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APPLICATION OF THE CLONAL SELECTION ALGORITHM FOR RECONSTRUCTION OF THE THIRD KIND BOUNDARY CONDITION

Summary. In this paper the inverse heat conduction problem with the third kind boundary condition is solved by using the Clonal Selection Algorithm (CSA) – the heuristic algorithm imitating the rules of functioning of immunological system in the mammals bodies. Solution of investigated problem consists in identifying the unknown heat transfer coefficient and reconstructing the distribution of state function. To achieve this goal a procedure based on minimization of the appropriate functional realized by the aid of CSA algorithm is elaborated.

ZASTOSOWANIE ALGORYTMU SELEKCJI KLONALNEJ DO ODTWORZENIA WARUNKU BRZEGOWEGO TRZECIEGO RODZAJU

Streszczenie. Celem niniejszej pracy jest rozwiązanie zadania przewodnictwa ciepła z warunkiem brzegowym trzeciego rodzaju przy zastosowaniu algorytmu selekcji klonalnej (CSA) – algorytmu heurystycznego naśladującego reguły funkcjonowania układu immunologicznego ssaków. Rozwiązanie badanego zagadnienia polega na identyfikacji nieznanego współczynnika wnikania ciepła oraz rozkładu funkcji stanu. Aby osiągnąć ten cel opracowana została procedura oparta na minimalizacji odpowiedniego funkcjonału, realizowana przy użyciu algorytmu CSA.

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1. Introduction

The inverse heat conduction problem means a heat conduction problem with the incomplete mathematical description, consisting in determination of the function describing the distribution of temperature and reconstruction of some of the boundary conditions [1, 17]. The inverse problem is much more difficult for solving than the direct heat conduction problem in which the initial and boundary conditions are known, only the temperature needs to be found. However, some methods for solving the inverse problem are proposed, like for example the boundary elements method [12], mollification method [15], Monte Carlo method [6], homotopy perturbation method [7], methods applying the wavelets theory [18] or genetic algorithms [13,21] and others (see for example [4,14,16]). In papers [8–10] the authors have used the algorithms of swarm intelligence based on the intelligent behavior of the swarm resulting from the cooperation of many simple individuals building the common solution of the problem by finding independently only a small piece of the solution. The ant and bees algorithms have been used for minimizing a functional, representing the differences between the known exact and sought approximate solutions and forming a crucial part of the approach.

In this paper we consider a similar approach, however based on the Clonal Selection Algorithm [2,19]. CSA belongs to the group of immune algorithms basing on the analogy between functioning of the immune system in the mammals bodies and the problem of finding the optimal solution of some problem. Goal of the immune system is the creation of antibodies eliminating the foreign harmful organisms called antigens. The more similar is the antibody to the antigen, the more efficiently the antibody works. The measure of the immune algorithm effectiveness can be minimization of the difference between the pattern (antigen) and antibody. Therefore in solving an optimization problem the sought optimal solution plays the role of antigen, whereas the values of objective function in obtained partial solutions can be considered as the antibodies. The immune algorithms appeared to be a useful tool for data analysis and for solving the combinatorial optimization problems as well as technical problems of different kind [3, 11, 23].

Described algorithm will be applied for solving the inverse heat conduction problem with boundary condition of the third kind which means for determination of the temperature distribution and reconstruction of the form of heat transfer coefficient appearing in the boundary condition of the third kind. Problem of identifying the heat transfer coefficient was already investigated, for example in [5, 20, 22], however the idea of using CSA for solving this problem is new.

2. Clonal Selection Algorithm

Clonal Selection Algorithm runs according to the following steps [19].

1. We randomly generate the initial population composed of N vectors. Elements of the vectors are calculated from the following relation

$$x_j = x_{j,lo} + R\,(x_{j,up} - x_{j,lo}), \quad j = 1, \ldots, n, \tag{1}$$

   where $x_{j,lo}$ and $x_{j,up}$ denote the lower bound and upper bound, respectively, of the range of variable x_j and R is the random variable of uniform distribution selected from the range $[0, 1]$.

2. Obtained vectors are sorted with respect to the non-increasing values of objective function. Position i_{rank} of a vector denotes its rating position.

3. Part of the population, composed of $c{\cdot}N$ elements where $c < 1$ is a parameter of algorithm, is cloned. It means, for each of $c \cdot N$ solutions some number of copies N_c is produced. Number of copies depends on the rating position and is expressed by relation

$$N_c^i = \left\lfloor \frac{\beta\, N}{i_{rank}} \right\rfloor, \quad i \in \{1, \ldots, \lfloor c\, N \rfloor\}, \tag{2}$$

   where β is the maximal number of copies parameter, i_{rank} describes the rating position of ith solution and $\lfloor\cdot\rfloor$ denotes the integer part.

4. Clones are exposed to the process of "maturation". To each copy the hypermutation is applied which consists in random modification of the values of all the variables. Elements of the vectors of independent variables are calculated by using the following relation

$$x_j^* = x_j + p\,(x_{j,up} - x_{j,lo}) \cdot G(0, 1), \quad j = 1, \ldots, n, \tag{3}$$

   where p is the range of mutation and $G(0, 1)$ is the random variable of Gaussian distribution of mean value equal to 0 and standard deviation equal to 1.

   Range p of mutation is the smaller the better is the adaptation of solution. Thus, at the beginning of process the maximal range p_{max} of mutation should be given and during the process the value of p changes according to the relation

$$p = p_{max} \exp\left(\hat{\rho}\, \frac{t}{t_{max}}\right), \tag{4}$$

where t denotes the number of generation, t_{max} describes the maximal number of generations and $\hat{\rho}$ is a mutation parameter of the algorithm.

Range of mutation of the solution placed on the ith rating position depends on its adaptation, it means on its rating position, and is expressed by formula

$$p_i = p \frac{f_i - k\, f_b}{f_w - k\, f_b}, \tag{5}$$

where f_i, f_b and f_w denote adaptations of the solution placed on the ith rating position, the best one and the worst one from the set of cloned solutions, respectively, and k is a coefficient denoting the ratio of the hypothetic adaptation of optimal solution and the adaptation of the best solution in current generation.

5. After applying the hypermutation, the value of objective function for the new solution is calculated. If the mutated solution is better than the original one, it replaces the original one. In consequence, the best one among N_c^i solutions move to the new generation.

6. The other, not cloned part of population, composed of $N_d = N - c \cdot N$ elements, is in each iteration replaced by new, randomly generated solutions. Thanks to this, not examined yet part of domain can be investigated.

3. Formulation of the problem

We consider the problem described by heat conduction equation of the form

$$c\, \rho \frac{\partial u}{\partial t}(x,t) = \lambda \frac{\partial^2 u}{\partial x^2}(x,t), \qquad x \in [0,d], \quad t \in [0,T] \tag{6}$$

with the following initial and boundary conditions

$$u(x,0) = u_0, \qquad x \in [0,d], \tag{7}$$

$$\frac{\partial u}{\partial x}(0,t) = 0, \qquad t \in [0,T], \tag{8}$$

where c is the specific heat, ρ denotes the mass density, λ is the thermal conductivity and u, t and x refer to the temperature, time and spatial location. On boundary for $x = d$ the boundary condition of the third kind is assumed

$$-\lambda \frac{\partial u}{\partial x}(d,t) = \alpha \,(u(d,t) - u_\infty), \qquad t \in [0,T], \tag{9}$$

where u_∞ describes the temperature of environment and α denotes the heat transfer coefficient, values of which is sought. Another element, which should be found, is the distribution of temperature $u(x,t)$ in considered domain. If the value α of heat transfer coefficient is assumed as known, the problem, defined by equations (6)-(9), becomes the direct heat conduction problem which can be solved by using one of the known method (for example, the finite difference method). Thus, the received approximate solution $\tilde{u}(x_i, t_j)$ in the nodes of mesh depends on this specifically assumed value of α.

Next, we determine the optimal value of α by minimizing the following functional

$$P(\alpha) = \sqrt{\sum_{j=1}^{m} \big(u(d,t_j) - \tilde{u}(d,t_j)\big)^2} \tag{10}$$

representing the differences between obtained results u and given values $\tilde{u}$ on the boundary for $x = d$ where the boundary condition is reconstructed. For minimizing functional (10) the CSA algorithm is used. It is important to realize that each determination of the value of functional (10) requires to solve the appropriate direct heat conduction problem.

4. Numerical example

Described approach will be tested by an example in which $c = 1000\,[\mathrm{J/(kg\cdot K)}]$, $\rho = 2679\,[\mathrm{kg/m^3}]$, $\lambda = 240\ [\mathrm{W/(m\cdot K)}]$, $T = 1000\,\mathrm{s}$, $d = 1\,\mathrm{m}$, $u_0 = 1013\,\mathrm{K}$ and $u_\infty = 298\,\mathrm{K}$. We need to identify the value of heat transfer coefficient, exact value of which is known: $\alpha = 28\,[\mathrm{W/(m^2\cdot K)}]$. For constructing functional (10) we use the exact values of temperature, determined for the given α, and values noised by the random error of 2% and 5%. The measurement point is located on the boundary for $x = 1$ and in the 5% and 10% distance away from this boundary. And the CSA algorithm is executed for the following values of parameters: number of individuals in one population $N = 10$, part of cloned population $c = 0.9$, maximal number of copies parameter $\beta = 1.5$, maximal range of mutation $p_{max} = 10$, coefficient of the mutation range $\hat{\rho} = -0.2$, coefficient of adaptation ratio $k = 0.95$, maximal number of generations (number of iterations) $t_{max} = 100$ and range of each variable $[-500, 500]$.

Figure 1 presents the distributions of relative error in α reconstruction in dependence on the number of iterations, calculated for the control point located, respectively, in point $x = 1$, $x = 0.95$ or $x = 0.9$, for input data burdened by 2% error in each case. Similar combination of relative error distributions, but for 5% perturbation of input data, is displayed in Figure 2. Quality of parameter α reconstruction depends certainly on the location of control point, however, we may observe that only few iterations of the procedure is needed to receive the reconstruction of parameter α with the relative error smaller than the input data error. We may also notice that, although the satisfying approximations can be obtained quickly, the successive iterations does not improve significantly the results. That is why the proposed procedure can be used for determining the starting point for the more traditional numerical method of solving such kind of problems, which requires a good starting point and maybe will give the possibility to improve the final results.

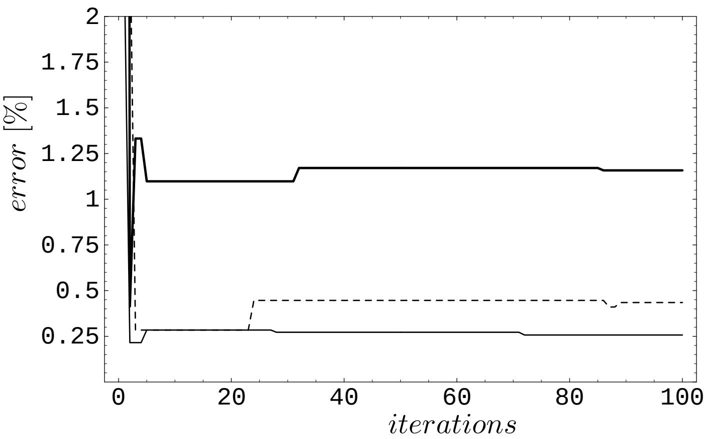

Fig. 1. Distributions of relative error in α reconstruction in dependence on the number of iterations, calculated for the control point located in point $x = 1$ (normal line), $x = 0.95$ (dashed line) or $x = 0.9$ (bold line) for input data burdened by 2% error

Rys. 1. Rozkłady błędu względnego odtworzenia α w zależności od liczby iteracji, dla punktu kontrolnego zlokalizowanego w punkcie $x = 1$ (linia zwykła), $x = 0.95$ (linia przerywana) i $x = 0.9$ (linia pogrubiona), dla danych wejściowych zaburzonych błędem 2%

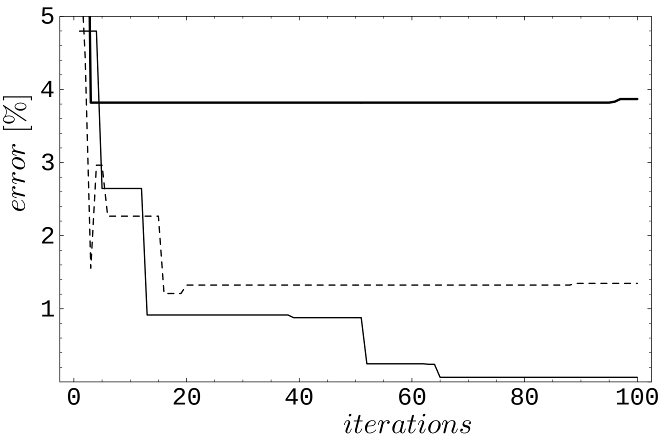

Fig. 2. Distributions of relative error in α reconstruction in dependence on the number of iterations, calculated for the control point located in point $x = 1$ (normal line), $x = 0.95$ (dashed line) or $x = 0.9$ (bold line) for input data burdened by 5% error

Rys. 2. Rozkłady błędu względnego odtworzenia α w zależności od liczby iteracji, dla punktu kontrolnego zlokalizowanego w punkcie $x = 1$ (linia zwykła), $x = 0.95$ (linia przerywana) i $x = 0.9$ (linia pogrubiona), dla danych wejściowych zaburzonych błędem 5%

Reconstruction of the temperature distribution even for the worst location of control point is very good – the calculated values of temperature in the boundary for $x = 1$, where the boundary condition is reconstructed, are very close to the exact values which is showed in Figures 3 and 4. And finally, in Table 1 the absolute and relative errors of parameter α and temperature u reconstruction, obtained for various input data errors and various locations of control point, are displayed which confirm the above conclusions.

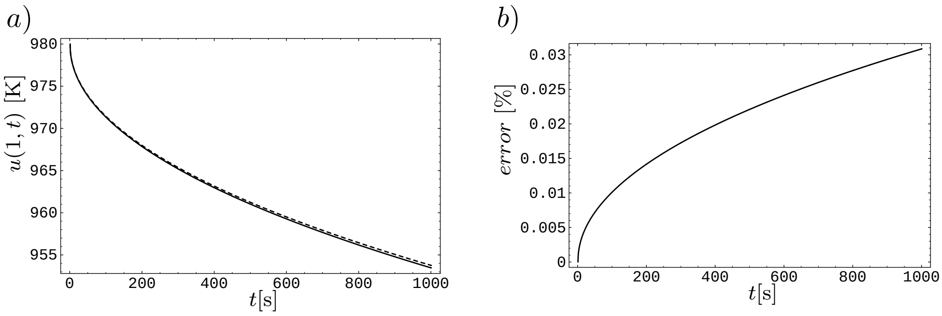

Fig. 3. Distribution of temperature $u(x,t)$ for $x = 1$ (solid line – exact solution, dashed line – reconstructed values) obtained for input data noised by 2% error and control point located in point $x = 0.9$ (a) and error of this reconstruction (b)

Rys. 3. Rozkład temperatury $u(x,t)$ na brzegu $x = 1$ (linia ciągła – wartości dokładne, linia przerywana – wartości odtworzone) dla danych wejściowych zaburzonych 2% błędem) i punktu kontrolnego zlokalizowanego w punkcie $x = 0.9$ (a) oraz błąd tego odtworzenia (b)

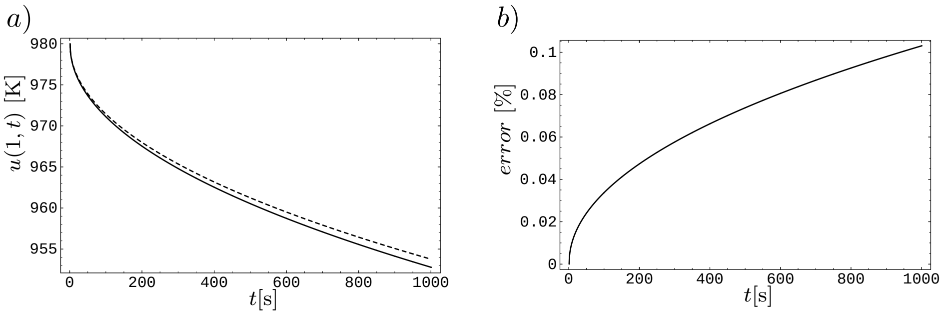

Fig. 4. Distribution of temperature $u(x,t)$ for $x = 1$ (solid line – exact solution, dashed line – reconstructed values) obtained for input data noised by 5% error and control point located in point $x = 0.9$ (a) and error of this reconstruction (b)

Rys. 4. Rozkład temperatury $u(x,t)$ na brzegu $x = 1$ (linia ciągła – wartości dokładne, linia przerywana – wartości odtworzone) dla danych wejściowych zaburzonych 5% błędem) i punktu kontrolnego zlokalizowanego w punkcie $x = 0.9$ (a) oraz błąd tego odtworzenia (b)

5. Conclusions

Aim of this paper was to present the procedure for solving the inverse heat conduction problem with boundary condition of the third kind. For solving this problem we needed to identify the heat transfer coefficient and to reconstruct

Table 1

Absolute and relative errors of parameter α and temperature u reconstruction

x	noise [%]	Δ_α [%]	δ_α [K]	Δ_u [%]	δ_u [K]
1	0	7.17×10^{-5}	2.01×10^{-5}	1.91×10^{-6}	1.82×10^{-5}
	2	0.257	0.072	0.007	0.065
	5	0.062	0.018	0.002	0.016
0.95	0	5.43×10^{-5}	1.52×10^{-5}	1.45×10^{-6}	1.38×10^{-5}
	2	0.435	0.122	0.012	0.11
	5	1.347	0.377	0.036	0.343
0.90	0	2.16×10^{-4}	6.05×10^{-6}	5.77×10^{-6}	5.50×10^{-5}
	2	1.579	0.324	0.031	0.294
	5	3.869	1.083	0.103	0.983

the temperature distribution in considered region. Proposed approach consisted in computing the solutions of associated direct problems for given values of sought coefficient and determining its optimal value by comparing obtained solutions with the known exact solution. For minimizing functional expressing differences between approximate and exact solutions the Clonal Selection Algorithm was used.

Summing up, the proposed approach constitutes the effective tool, with respect to the velocity of working and the precision of obtained results, for solving inverse problems of considered kind. In each investigated case of input data the reconstruction errors, in α reconstruction as well as in temperature reconstruction, were smaller than the input data perturbations and the numbers of iterations needed for receiving the satisfying results were relatively small which makes the procedure useful and efficient.

References

1. Beck J.V., Blackwell B., St.Clair C.R.: *Inverse Heat Conduction: Ill Posed Problems*. Wiley Intersc., New York 1985.
2. Campelo F., Guimarães G., Igarashi H., Ramirez J.A.: *A clonal selection algorithm for optimization in electromagnetics*. IEEE Trans. on Magnetics **41** (2005), 1736-1739.

3. Dasgupta D.: *Artificial Immune Systems and their Applications*. Springer Verlag, New York 1998.
4. Grysa K., Leśniewska R.: *Different finite element approaches for inverse heat conduction problems*. Inverse Probl. Sci. Eng. **18** (2010), 3–17.
5. Grzymkowski R., Słota D.: *Numerical calculations of the heat-transfer coefficient during solidification of alloys*. Moving boundaries VI. Computational modelling of free and moving boundary problems, B. Sarler, C.A. Brebbia, eds., WIT Press, Southampton 2001, 41–50.
6. Haji-Sheikh A., Buckingham F.P.: *Multidimensional inverse heat conduction using the Monte Carlo method*. Trans. of ASME, Journal of Heat Transfer **115** (1993), 26–33.
7. Hetmaniok E., Nowak I., Słota D., Wituła R.: *Application of the homotopy perturbation method for the solution of inverse heat conduction problem*. Int. Comm. Heat & Mass Transf. **39** (2012), 30–35.
8. Hetmaniok E., Słota D., Zielonka A.: *Solution of the inverse heat conduction problem by using the ABC algorithm*. Lect. Notes Comput. Sc. **6086** (2010), 659–668.
9. Hetmaniok E., Słota D., Zielonka A.: *Application of the ant colony optimization algorithm for reconstruction of the thermal conductivity coefficient*. Lect. Notes Comput. Sc. **7269** (2012), 240–248.
10. Hetmaniok E., Słota D., Zielonka A., Wituła R.: *Comparison of ABC and ACO algorithms applied for solving the inverse heat conduction problem*. Lect. Notes Comput. Sc. **7269** (2012), 249–257.
11. Kilic O., Nguyen Q.M.: *Application of artificial immune system algorithm to electromagnetics problems*. Progress in Electromagnetics Research B **20** (2010), 1–17.
12. Kurpisz K., Nowak A.J.: *Numerical analysis of inverse heat conduction problems with Boundary Elements Method and combined techniques*. ZAMM Z. Angew. Math. Mech., Annual GAMM Conference 1993, 301–308.
13. Mera N.S., Elliott L., Ingham D.B.: *A multi-population genetic algorithm approach for solving ill-posed problems*. Comput. Mech. **33** (2004), 254–262.
14. Mochnacki B., Majchrzak E.: *Identification of macro and micro parameters in solidification model*. Bull. Pol. Acad. Sci. Tech. Sci. **55** (2007), 107–113.
15. Mourio D.A.: *The Mollification Method and the Numerical Solution of Ill-posed Problems*. John Wiley and Sons Inc., New York 1993.

16. Nowak I., Smolka J., Nowak A.J.: *An effective 3-D inverse procedure to retrieve cooling conditions in an aluminum alloy continuous casting problem.* Appl. Thermal Eng. **30** (2010), 1140–1151.
17. Özisik M.N., Orlande H.R.B.: *Inverse heat transfer: fundamentals and applications.* Taylor & Francis, New York 2000.
18. Qiu C.Y., Fu C.L., Zhu Y.B.: *Wavelets and regularization of the sideways heat equation.* Comput. Math. Appl. **46** (2003), 821–829.
19. Rudeński A.: *Zastosowanie algorytmu immunologicznego do optymalizacji silników indukcyjnych.* Prace Nauk. Inst. Maszyn, Napędów i Pomiarów Elektrycznych Pol. Wroc. **62** (2008), 144–149.
20. Ryfa A., Białecki R.: *The heat transfer coefficient spatial distribution reconstruction by an inverse technique.* Inverse Probl. Sci. Eng. **19** (2011), 117–126.
21. Słota D.: *Solving the inverse Stefan design problem using genetic algorithm.* Inverse Probl. Sci. Eng. **16** (2008), 829–846.
22. Słota D.: *Restoring boundary conditions in the solidification of pure metals.* Comput. & Structures **89** (2011), 48–54.
23. Timmis J., Neal M., Hunt J.: *An artificial immune system for data analysis.* Biosystems **55** (2000), 143–150.

Omówienie

Celem niniejszej pracy jest prezentacja procedury wykorzystywanej do rozwiązania odwrotnego zagadnienia przewodnictwa ciepła z warunkiem brzegowym trzeciego rodzaju. Rozwiązanie badanego zagadnienia polega na odtworzeniu wartości współczynnika wnikania ciepła oraz rozkładu temperatury w rozważanym obszarze. Proponowane w pracy podejście to rozwiązanie bezpośrednich zagadnień przewodnictwa ciepła dla ustalonych wartości szukanego współczynnika i wyznaczenie jego optymalnej wartości poprzez porównywanie uzyskanych przybliżonych rozkładów temperatury z zadanym temperaturami w punktach pomiarowych. Do minimalizacji funkcjonału wyrażającego tę różnicę wykorzystany został algorytm selekcji klonalnej.

Podsumowując, proponowane podejście stanowi efektywne narzędzie, pod względem szybkości działania i dokładności uzyskanych wyników, do rozwiązywania zagadnień odwrotnych rozważanego typu. W każdym badanym przypadku

danych wejściowych uzyskane błędy odtworzeń, zarówno dla odtwarzanego parametru α, jak i odtwarzanej temperatury, są mniejsze niż zaburzenia danych wejściowych, a liczba iteracji potrzebna do uzyskania satysfakcjonujących wyników jest stosunkowo mała, co czyni zaproponowaną procedurę wygodną i efektywną.