ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 2011

Seria: MATEMATYKA STOSOWANA z. 1 Nr kol. 1854

Anna KORCZAK

Institute of Mathematics
Silesian University of Technology

APPLICATION OF THE HAAR AND B-SPLINE WAVELETS TO APPROXIMATE SOLUTION OF THE BOUNDARY PROBLEMS

Summary. This paper presents an application of the wavelet theory to function approximation and to approximate solutions of chosen differential equations. There are considered the functions of class $L^2(\mathbb{R})$ and the first and the second order linear differential equations in interval $[0, 1]$ with appropriate boundary conditions. Haar wavelet and linear B-spline wavelet were applied to approximate these functions and solving these problem.

ZASTOSOWANIE FALEK HAARA I B-SPLAJNOWEJ W PRZYBLIŻONYM ROZWIĄZYWANIU ZAGADNIEŃ BRZEGOWYCH

Streszczenie. W artykule przedstawiono zastosowanie teorii falek w aproksymacji funkcji oraz w aproksymacji rozwiązań wybranych równań różniczkowych. Rozważane są również funkcje klasy $L^2(\mathbb{R})$ oraz równania różniczkowe liniowe rzędów pierwszego i drugiego, określone na przedziale $[0, 1]$, z odpowiednimi warunkami brzegowymi. Do aproksymacji wyżej wymienionych funkcji oraz równań zostały zastosowane falka Haara oraz falka B-splajnowa liniowa.

2010 Mathematics Subject Classification: 80M99, 65M99.
Wpłynęło do Redakcji (received): 05.07.2011 r.

1. Introduction

Wavelet theory is a respectively new field of mathematics. It has been very intensively developed since eighties and is promising in usage to various applications. Wavelets find an application in many different problems like analyzing acoustic emission pulses [1], technical diagnostics [4], local images filtering [6], finite elements method [3]. Their popularity is caused by the combination of theoretical and applied mathematics represented by this approach.

Wavelet is a kind of function $\Psi(x) \in L^2(\mathbb{R})$ such that the set

$$B_\Psi = \{2^{j/2}\Psi(2^j x - k); j \in \mathbb{Z}, k \in \mathbb{Z}\} \tag{1}$$

is a base in space $L^2(\mathbb{R})$ [2]. Familly B_Ψ is called the wavelet base.

Previous definition implies that any function $f(x) \in L^2(\mathbb{R})$ can be showed as linear combination of base functions $\Psi_{jk} = 2^{j/2}\Psi(2^j x - k)$ (if Ψ is a wavelet). That means that function f can be written as the following series

$$f(x) = \sum_{j\in\mathbb{Z}} \sum_{k\in\mathbb{Z}} f_{jk}\Psi_{jk}(x), \tag{2}$$

where f_{jk} are the Fourier coefficients defined as follows

$$f_{jk} = \langle f(x), \Psi_{jk}(x)\rangle = \int_{-\infty}^{\infty} f(x)\Psi_{jk}(x)dx.$$

2. Haar wavelet

The simplest example of wavelet is the Haar wavelet which is defined as follows

$$H(x) = \begin{cases} 1, & x \in [0, \frac{1}{2}), \\ -1, & x \in [\frac{1}{2}, 1), \\ 0, & x \notin [0, 1). \end{cases}$$

Function $H(x)$ is connected with the set

$$B_H = \{2^{j/2}H(2^j - k); j \in \mathbb{Z}, k \in \mathbb{Z}\}. \tag{3}$$

Functions $H_{jk}(x) \in B_H$ (for fixed j and k) have the form

$$H_{jk}(x) = \begin{cases} 2^{j/2}, & x \in [k2^{-j}, (k+\frac{1}{2})2^{-j}), \\ -2^{j/2}, & x \in [(k+\frac{1}{2})2^{-j}, (k+1)2^{-j}), \\ 0, & x \notin [k2^{-j}, (k+1)2^{-j}). \end{cases} \tag{4}$$

The support of function $H_{jk}(x)$ is the interval

$$\operatorname{supp} H_{jk}(x) = \left[k2^{-j}, (k+1)\, 2^{-j}\right]. \tag{5}$$

Basic property of the Haar wavelet is the orthogonality of base H_{jk}

$$\langle H_{j_1k_1}, H_{j_2k_2} \rangle = 0 \quad \text{for} \quad j_1 \neq j_2 \quad \vee \quad k_1 \neq k_2. \tag{6}$$

Any function H_{jk} is obtained by translating and scaling of the Haar wavelet.

2.1. Haar wavelet approximation

Function $f(x)$ belonging to the appropriate function space can be approximated by using the Haar wavelet and the base B_H associated with it.

We approximate function f only in interval $[-2^{-m}, 2^{-m}]$, out of it we take value 0. Moreover, we define the value $r \in \mathbb{Z}$, which is the maximum approximation (resolution) level of function f.

For fixed m and r the approximation of function f is as follows

$$\tilde{f}(x) = f^m(x) + \sum_{j=m}^{r} \sum_{k=-2^{j-m}}^{2^{j-m}-1} f_{jk} H_{jk}(x). \tag{7}$$

where, for $j < m$, we have

$$f^m(x) = \begin{cases} 2^m \displaystyle\int_0^{2^{-m}} f(x)dx, & x \in [-2^{-m}, 0), \\ 2^m \displaystyle\int_{-2^{-m}}^{0} f(x)dx, & x \in [0, 2^{-m}). \end{cases}$$

Example 2.1

In this example, we will approximate function $f(x)$, defined as follows

$$f(x) = \begin{cases} \sin x + 0.5, & x \in [0, 1), \\ 0, & x \notin [0, 1). \end{cases}$$

Function $f(x)$ is the element of space $L^2(\mathbb{R})$ and can be approximated by using the Haar wavelet.

For the following approximation levels: $r = 1, 2, 3, 4$, Figure 1 illustrates the comparisons of function $f(x)$ and its approximations. Whereas, the error of approximations for levels: $r = 1, 2, 3, 4$, are displayed in Figure 2. Basing on the

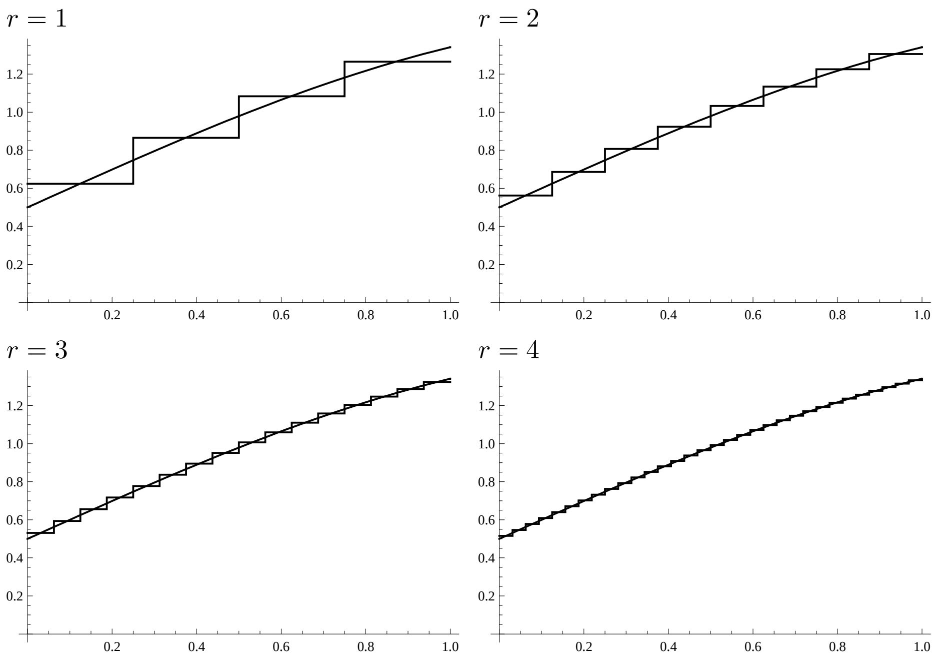

Fig. 1. Comparison of function $f(x)$ and its approximations
Rys. 1. Porównanie przebiegu funkcji $f(x)$ i jej aproksymacji

presented diagrams we can say that the increase of approximation level causes the double decrease of the approximation error.

Calculating the approximation error in points $x_i = \frac{i}{10}$ for $i = 0, 1, 2, \ldots, 10$ the efficiency of function approximation can be compared for different wavelets. Errors in points x_i for the Haar wavelet are shown in Table 1.

Wavelet base B_H gives an opportunity to approximate the solutions of linear differential equations. We consider differential equation of the form

$$y' + f(x)y = g(x), \tag{8}$$

for $x \in [0, 1]$, with the initial condition

$$y(0) = y_0,$$

where the unknown function $y = y(x)$ and the given functions $f(x)$ i $g(x)$ belong to the appropriate class in $[0, 1]$. By using wavelet base B_H we are looking for the solution having the following form

$$\tilde{y}_r(x) = a + \sum_{j=0}^{r} \sum_{k=0}^{2^j-1} a_{jk} H_{jk}(x), \tag{9}$$

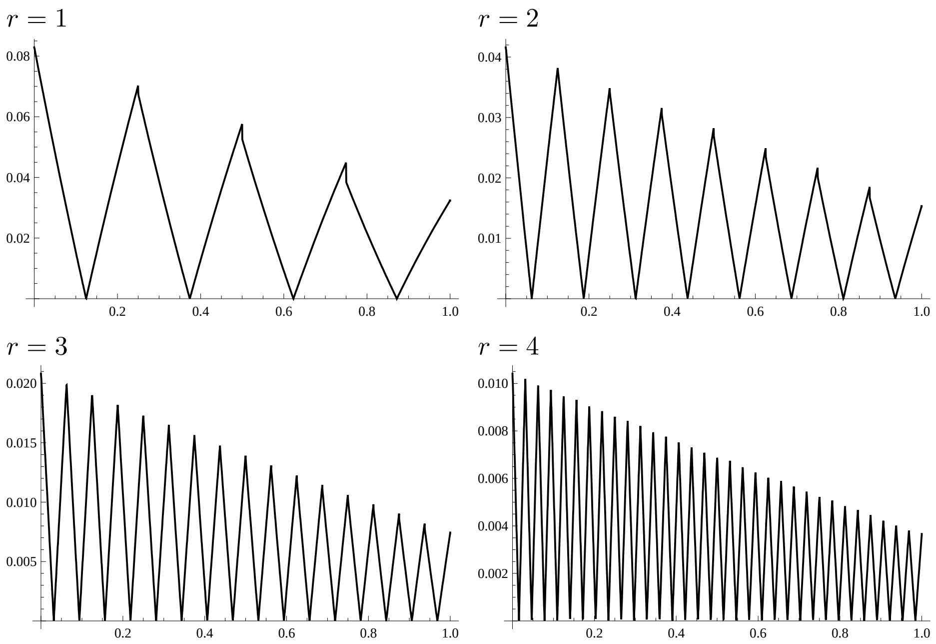

Fig. 2. Error of the function approximation
Rys. 2. Błąd aproksymacji funkcji

Function $\tilde{y}_r$ depends on parameters a and a_{jk}, which are unknown and need to be found. This function is constant in each of intervals $\left[i2^{-r-1}, (i+1)\, 2^{-r-1}\right)$, $i = 0, \dots, 2^{r+1} - 1$ [2].

Example 2.2

By using the wavelet base B_H we will find the approximate solution of the following differential equation of the first order

$$y'(x) + y(x) = x^2 \sin x, \qquad x \in [0, 1], \tag{10}$$

with initial condition

$$y(0) = 1.$$

Exact solution of this equation is the following

$$y(x) = -\frac{1}{2} e^{-x} (-3 + e^x \cos x - 2e^x x \cos x + e^x x^2 \cos x + e^x \sin x - e^x x^2 \sin x). \tag{11}$$

Table 1

Function approximation error

x_i	$r=1$	$r=2$	$r=3$	$r=4$
0	0.025	0.12	0.06	0.03
0.1	0.04	0.06	0.01	0.02
0.2	0.11	0.02	0.03	0.004
0.3	0.09	0.01	0.02	0.004
0.4	0.03	0.04	0.006	0.01
0.5	0.11	0.05	0.03	0.01
0.6	0.02	0.03	0.005	0.007
0.7	0.05	0.009	0.01	0.002
0.8	0.04	0.007	0.01	0.002
0.9	0.01	0.02	0.003	0.005
1.0	0.06	0.03	0.01	0.006

By using described algorithm of finding approximate solution we obtain the approximate solution in points

$$x_i = (i + \frac{1}{2})2^{-r-1}$$

for $i = 0, \ldots, 2^{r+1} - 1$.

For the following approximation levels: $r = 1, 2, 3, 4$, Figure 3 illustrates comparison of the exact solution of considered problem and its approximations. Whereas, the approximation errors for these levels are illustrated in Figure 4. Basing on the presented diagrams we can say that the increase of approximation level causes the decrease of approximation error.

In the next example of wavelet base B_H application three boundary problems for linear differential equation of the second order will be considered. Let us consider the equation

$$y''(x) + f(x)y'(x) + g(x)y(x) = h(x), \qquad x \in [0, 1], \tag{12}$$

with the following boundary conditions:

$$\begin{cases} y(0) = y_0, \\ y'(0) = y'_0, \end{cases} \tag{13}$$

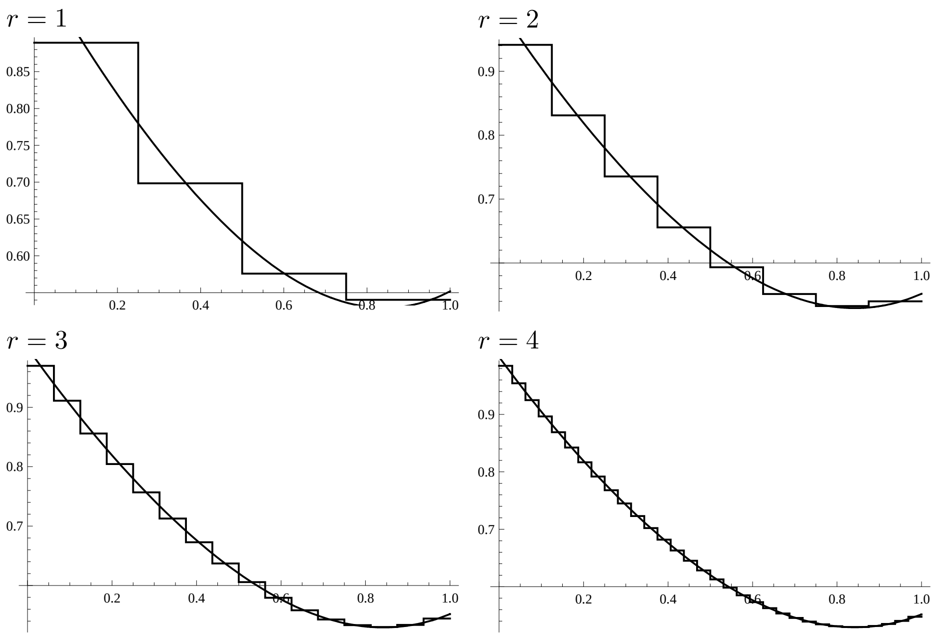

Fig. 3. Comparison of the exact and approximate solution of differential equation (10)
Rys. 3. Porównanie przebiegu rozwiązania dokładnego równania różniczkowego i jego aproksymacji (10)

$$\begin{cases} y(0) = y_0, \\ y(1) = y_1, \end{cases} \tag{14}$$

$$\begin{cases} y'(0) = y'_0, \\ y(1) = y_1. \end{cases} \tag{15}$$

The unknown function $y = y(x)$ and the given functions $f(x)$, $g(x)$ i $h(x)$ belong to the appropriate class.

Solution of this problem is sought as the sum (9). Similarly like in case of the first order equation the unknown elements are a and a_{jk}.

Example 2.3

By using the wavelet base B_H we will find the approximate solution of the following linear differential equation of the second order

$$y''(x) + y(x) = -\cos x\, e^x, \qquad x \in [0,1], \tag{16}$$

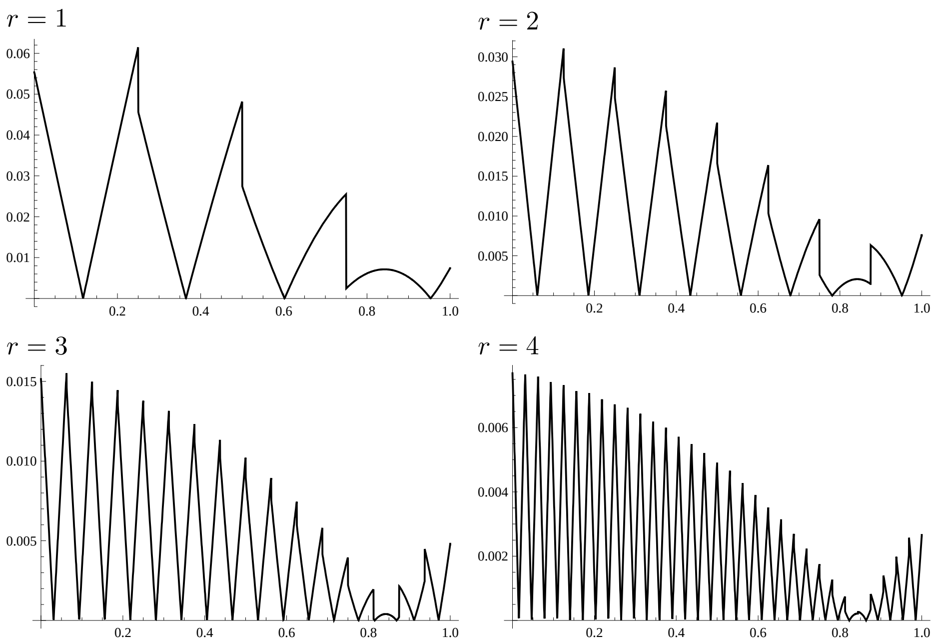

Fig. 4. Error of approximate solution of differential equation (10)
Rys. 4. Błąd aproksymacji rozwiązania równania różniczkowego (10)

with the boundary conditions of the second type

$$y(0) = 0$$

and

$$y(1) = 0.$$

Exact solution of this equation is the following:

$$\begin{aligned} y(x) \quad = \quad & \frac{1}{10}(2\cos x - 2e^x \cos x \cos 2x + 5e \sin x - 5e^x \sin x + e \cos 2 \sin x \\ & -2 \operatorname{ctg} 1 \sin x + 2e \cos 2 \operatorname{ctg} 1 \sin x + 2e \sin 2 \sin x - e \operatorname{ctg} 1 \sin 2 \sin x \\ & +e^x \cos x \sin 2x - 2e^x \sin x \sin 2x). \end{aligned}$$

By applying the proposed approach we obtain the approximated solution in points

$$x_i = (i + \frac{1}{2})2^{-r-1}$$

for $i = 0, \ldots, 2^{r+1} - 1$. For the following approximation levels: $r = 1, 2, 3, 4$, Figure 5 shows the comparison of the exact and approximate solution of the considered

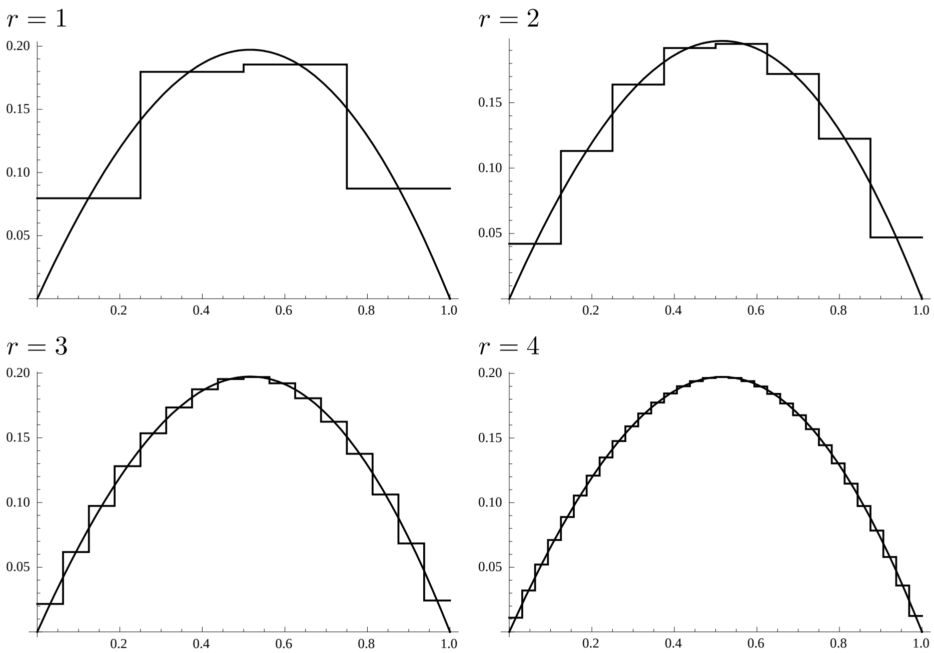

Fig. 5. Comparison solution of differential equation (16) and its approximation
Rys. 5. Porównanie przebiegu rozwiązania równania różniczkowego (16) i jego aproksymacji

problem. Whereas, approximation errors for levels: $r = 1, 2, 3, 4$, are illustrated in Figure 6. According to the presented diagrams we can say that with the increase of approximation level the approximation error decreases.

3. B-spline wavelet

B-spline wavelet is often defined by means of the scaling functions [7]. Scaling function of the m-order B-spline wavelet is defined as the following convolution:

$$\phi_m(t) = (\phi_{m-1} * \phi_1(t)) = \int_{-\infty}^{\infty} \phi_{m-1}(t-x)\phi_1(x)dx = \int_0^1 \phi_{m-1}(t-x)dx. \quad (17)$$

Moreover, the m-order B-spline wavelet can be defined as the linear combination of scaling functions:

$$\Psi_m(x) = \sum_{k=0}^{3m-2} q_k \phi_m(2x-k), \quad (18)$$

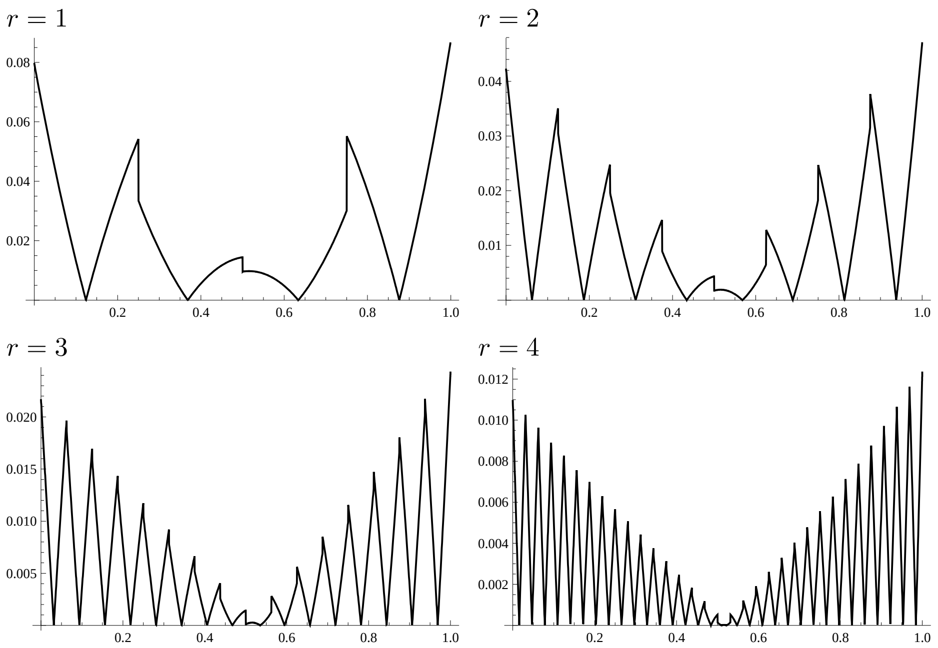

Fig. 6. Error of approximate solution of differential equation (16)
Rys. 6. Błąd aproksymacji rozwiązania równania różniczkowego (16)

where

$$q_k = (-1)^k \, 2^{1-m} \sum_{l=0}^{m} \binom{m}{l} \phi_{2m}(k+1-l). \tag{19}$$

It is worth to notice that the first order B-spline wavelet is the Haar wavelet.

Scaling function of the linear B-spline wavelet is defined by the formula

$$\phi_2(x) = \begin{cases} x, & x \in [0,1), \\ 2-x, & x \in [1,2), \\ 0, & x \notin [0,2). \end{cases} \tag{20}$$

Further scaling function for $m = 2$ is denoted as $\phi(x)$. Then, the two-scale relation for this function is the following

$$\phi_{j,k}(x) = \begin{cases} x_j - k, & x \in [k2^{-j}, (k+1)2^{-j}), \\ 2-(x_j-k), & x \in [(k+1)2^{-j}, (k+2)2^{-j}), \\ 0, & x \notin [k2^{-j}, (k+2)2^{-j}), \end{cases} \tag{21}$$

where $x_j = 2^j x - k$.

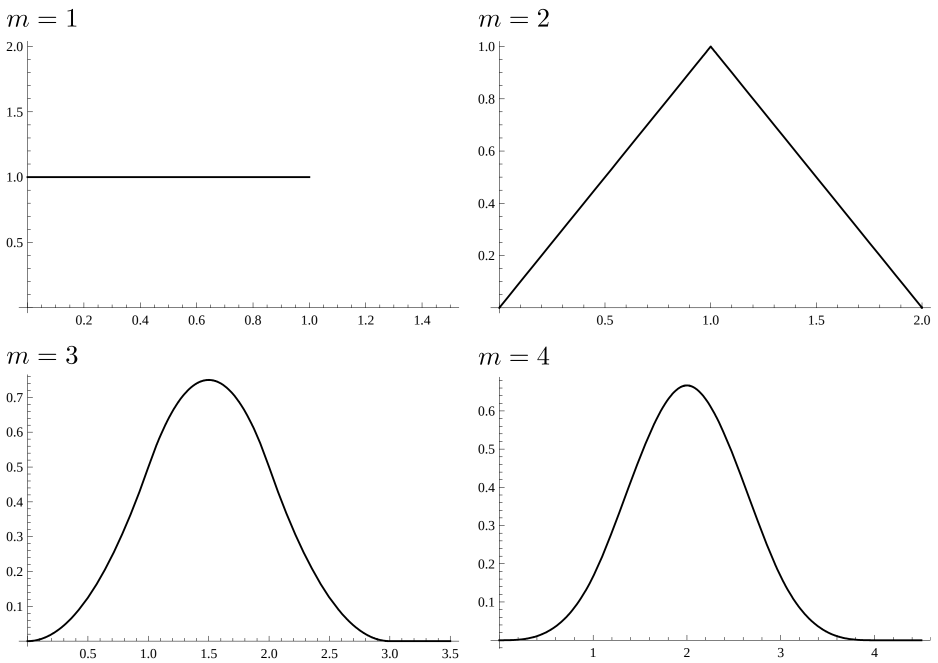

Fig. 7. Scaling functions for $m = 1, 2, 3, 4$
Rys. 7. Funkcje skalujące dla $m = 1, 2, 3, 4$

Function $\phi_{j,k}(x)$ is defined as the scaled and transformed function $\phi(x)$ by using parameters j and k. Functions $\phi_{j,k}(x)$ are helpful in forming the functions approximation.

Linear B-spline wavelet is defined by the formula:

$$\Psi_2(x) = \begin{cases} \frac{x}{6}, & x \in [0, \frac{1}{2}), \\ \frac{1}{6}(-7x+4), & x \in [\frac{1}{2}, 1), \\ \frac{1}{6}(16x-19), & x \in [1, \frac{3}{2}), \\ \frac{1}{6}(-16+29), & x \in [\frac{3}{2}, 2), \\ \frac{1}{6}(7x-17), & x \in [2, \frac{5}{2}), \\ \frac{1}{6}(-x+3), & x \in [\frac{5}{2}, 3), \\ 0, & x \notin [0, 3). \end{cases} \tag{22}$$

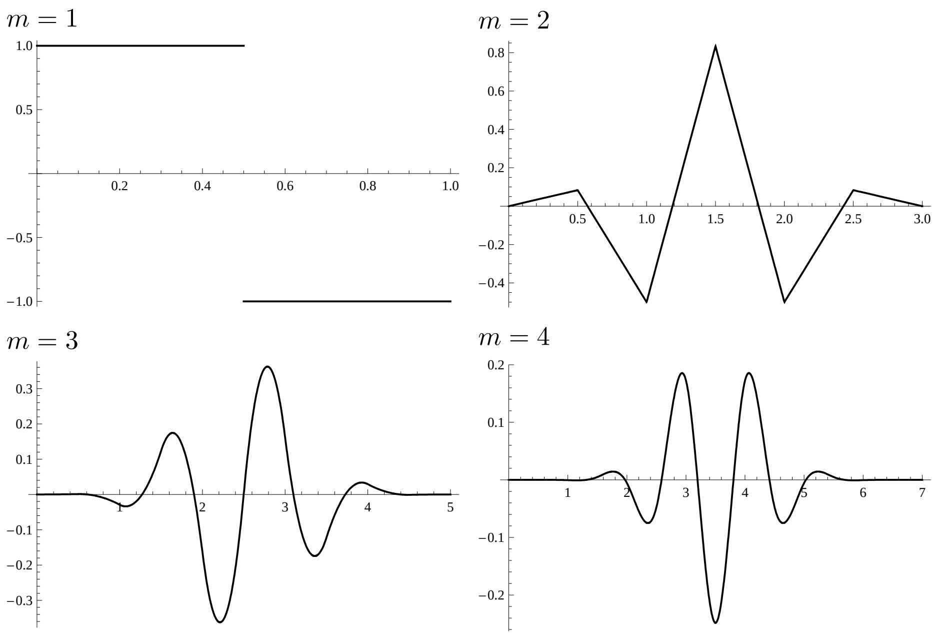

Fig. 8. B-spline wavelets of order $1, 2, 3, 4$
Rys. 8. Falki B-splajnowe rzędu $1, 2, 3, 4$

$\Psi_2(x)$ wavelet is associated with the wavelet base [5] ($x_j = 2^j x - k$):

$$\psi_{j,k}(x) = \begin{cases} x_j \frac{2^{j/2}}{6}, & x \in [k2^{-j}, (k+\frac{1}{2})2^{-j}), \\ (-7x_j + 4)\frac{2^{j/2}}{6}, & x \in [(k+\frac{1}{2})2^{-j}, (k+1)2^{-j}), \\ (16x_j - 19)\frac{2^{j/2}}{6}, & x \in [(k+1)2^{-j}, (k+\frac{3}{2})2^{-j}), \\ (-16x_j + 29)\frac{2^{j/2}}{6}, & x \in [(k+\frac{3}{2})2^{-j}, (k+2)2^{-j}), \\ (7x_j - 17)\frac{2^{j/2}}{6}, & x \in [(k+2)2^{-j}, (k+\frac{5}{2})2^{-j}), \\ (-x_j + 3)\frac{2^{j/2}}{6}, & x \in [(k+\frac{5}{2})2^{-j}, (k+3)2^{-j}), \\ 0, & x \notin [k2^{-j}, (k+3)2^{-j}]. \end{cases} \tag{23}$$

Similarly like in case of the Haar wavelet, parameter j is responsible for scaling and parameter k is responsible for translating the function ψ_{jk}.

3.1. Linear B-spline wavelet approximation [5]

For any fixed positive integer $r \in \mathbb{N}$, the function $f(x)$ defined over $[0,1]$ may be represented by the B-spline scaling functions in the following way

$$f(x) = \sum_{k=-1}^{2^{r+1}-1} s_k \phi_{r+1,k}(x), \tag{24}$$

where

$$s_k = \int_0^1 f(x)\tilde{\phi}_{r+1,k}(x)dx, \qquad k = -1, 0, \ldots, 2^{r+1}-1. \tag{25}$$

Functions $\tilde{\phi}_{r+1,k}(x)$ are the dual functions for $\phi_{r+1,k}(x)$ given by the relation

$$\int_0^1 \tilde{\Phi}_{r+1}\Phi_{r+1}dx = I, \tag{26}$$

where $\Phi_{r+1} = [\phi_{r+1,-1}, \phi_{r+1,0}, \ldots, \phi_{r+1,2^{r+1}-1}]^T$ and I is the identity matrix of dimension .$(2^{r+1}+1) \times (2^{r+1}+1)$
Let

$$P_{r+1} = \int_0^1 \Phi_{r+1}\Phi_{r+1}^T dx. \tag{27}$$

From (26) and (27) we get

$$\tilde{\Phi}_{r+1} = (P_{r+1})^{-1}\Phi_{r+1}. \tag{28}$$

Example 3.1

In this example we approximate the function $f(x)$ defined as follows

$$f(x) = \begin{cases} \sin x + 0.5, & x \in [0,1), \\ 0, & x \notin [0,1). \end{cases}$$

For the first approximation level the comparison of function $f(x)$ and its approximation is shown in Figure 9.

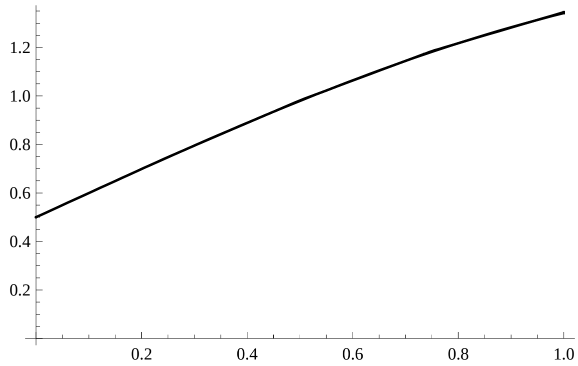

Fig. 9. Comparison of function $f(x)$ and its approximation for $r = 1$
Rys. 9. Porównanie przebiegu funkcji i jej aproksymacji dla $r = 1$

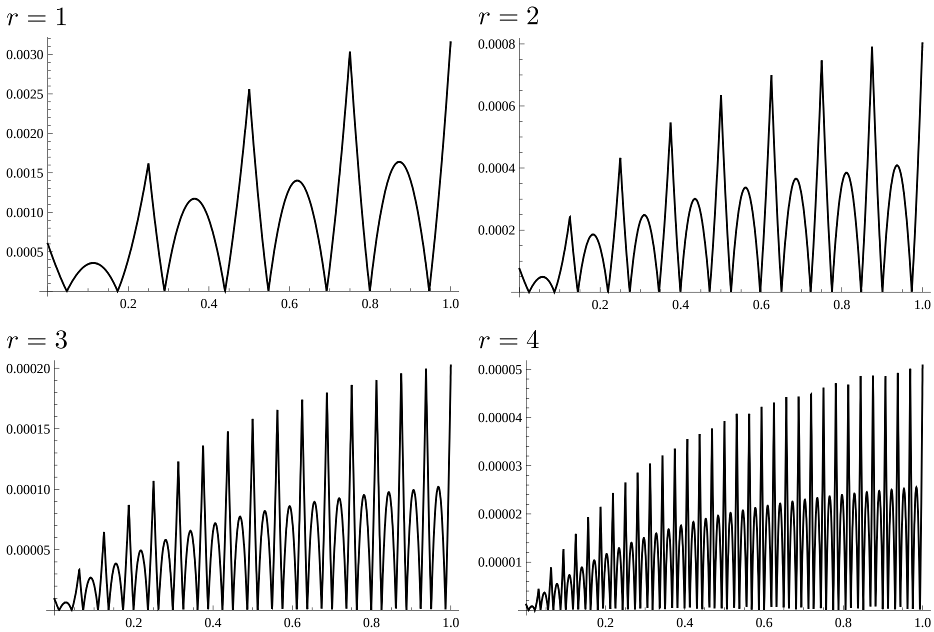

Fig. 10. Error of the function approximation
Rys. 10. Błąd aproksymacji funkcji

Diagram of $f(x)$ and its approximation are practically the same, even for first approximation level which confirms the good precision of approximation. Error analysis for the successive levels gives the opportunity to compare efficiency of approximation. Approximation errors for levels: $r = 1, 2, 3, 4$, are illustrated in Figure 10. Basing on the presented diagrams we can say that the increase of

Table 2

Error of the approximate solution

x_i	$r=1$	$r=2$	$r=3$	$r=4$
0	0.0006	0.00008	$9.4 \cdot 10^{-6}$	$1.2 \cdot 10^{-6}$
0.1	0.0003	0.00006	0.00002	$2.8 \cdot 10^{-6}$
02	0.0004	0.0001	$1.9 \cdot 10^{-6}$	$10 \cdot 10^{-6}$
0.3	0.0003	0.0002	$9.5 \cdot 10^{-6}$	$10 \cdot 10^{-6}$
0.4	0.001	0.00001	0.00006	$1.9 \cdot 10^{-6}$
0.5	0.003	0.0006	0.0002	0.00004
0.6	0.001	0.00005	0.00007	$1.3 \cdot 10^{-6}$
0.7	0.0003	0.0003	$4.7 \cdot 10^{-6}$	0.00002
0.8	0.00002	0.0003	$9.9 \cdot 10^{-6}$	0.00002
0.9	0.001	0.00002	0.00009	$2.2 \cdot 10^{-6}$
1.0	0.003	0.0008	0.0002	0.00005

approximation level causes the decrease of approximation error. Errors in points x_i for the B-spline wavelet are shown in Table 2.

3.2. B-spline wavelet approximation [5]

Let us consider the linear differential equation of the first order

$$y'(x) + f(x)y = g(x), \tag{29}$$

with the initial condition

$$y(0) = y_0, \tag{30}$$

where $f(x)$, $g(x)$ are the given functions of class $L^2[0,1]$, y_0 is the given real number and $y(x)$ is the unknown function. The differentiation of the vectors Φ_{r+1} and Ψ can be expressed in following way

$$\Phi'_{r+1} = D_\Phi \Phi_{r+1}, \qquad \Psi' = D_\Psi \Psi, \tag{31}$$

where D_Φ and D_Ψ denote the $(2^{r+1}+1) \times (2^{r+1}+1)$ operational matrices of derivative for B-spline scaling functions and wavelet.

Approximation of the function $f(x)$ can be expressed

$$y(x) = C^T \Psi(x), \tag{32}$$

where C is an unknown vector of dimension $2^{r+1} + 1$. From (32) we obtain

$$y'(x) = C^T \Psi'(x) = C^T D_\Psi \Psi(x). \tag{33}$$

Using (29) and (33) we have

$$C^T D_\Psi \Psi + f(x) C^T \Psi = g(x). \tag{34}$$

From (30) and (32) we get

$$C^T \Psi(0) = y_0. \tag{35}$$

To find the solution y(x) first we collocate equality (34) in $x_i = \frac{2i-1}{2^{r+1}-2}$, for $i = 1, 2, \ldots, 2^{r+1} - 1$. The resulting equation with initial condition (35) generates $2^{r+1} + 1$ linear equations. Solution of these equations is vector C which can be set into (32) and gives the approximation (29).

Example 3.2

By using wavelet base $\Psi_{j,k}$, we will find the approximate solution of the following differential equation

$$y'(x) + y(x) = x^2 \sin x, \qquad x \in [0, 1] \tag{36}$$

with the initial condition

$$y(0) = 1.$$

Diagram of $f(x)$ and its approximation are practically the same even for first approximation level (Figure 11) what means the good precision of approximation. Error analysis at the successive levels gives opportunity to compare the efficiency of approximations. Approximation errors for levels: $r = 1, 2, 3, 4$, are displayed in Figure 12. Basing on the presented diagrams we can say that with the increase of approximation level the approximation error decreases.

We consider now the second order linear differential equation

$$y''(x) + f(x)y' + g(x)y(x) = h(x), \tag{37}$$

with the boundary condition

$$y(0) = y_0, \qquad y(1) = y_1, \tag{38}$$

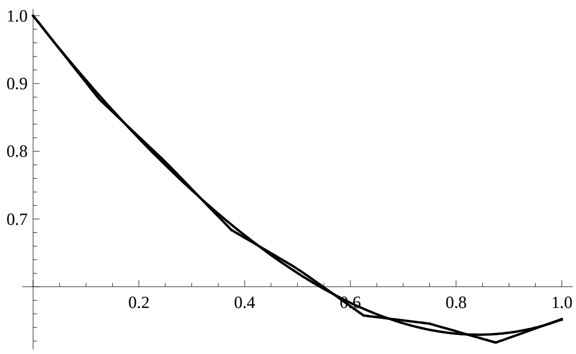

Fig. 11. Comparison of the solution of differential equation and its approximation for $r = 1$

Rys. 11. Porównanie przebiegu rozwiązania równania różniczkowego i jego aproksymacji dla $r = 1$

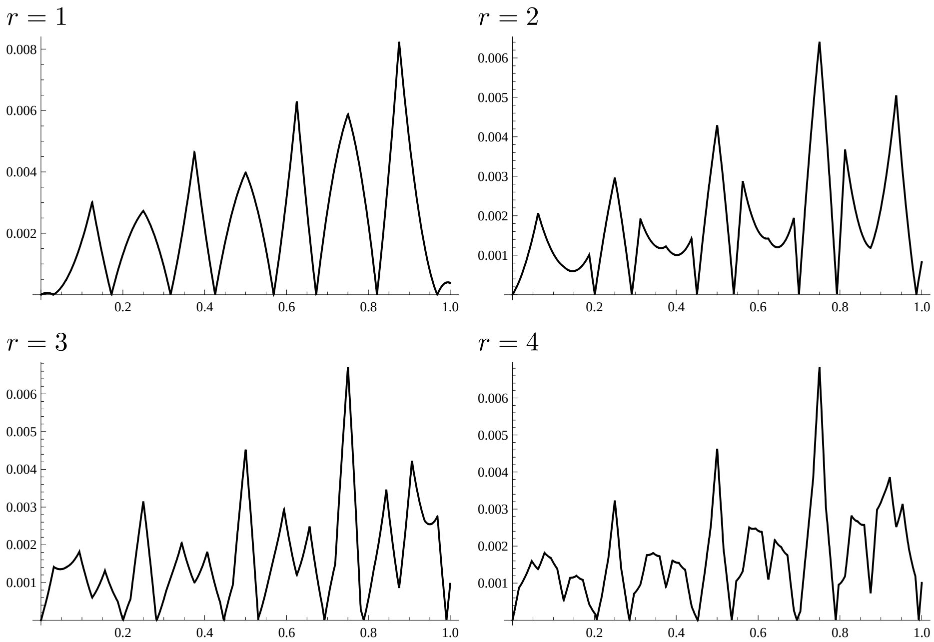

Fig. 12. Error of approximate solution of the differential equation

Rys. 12. Błąd aproksymacji rozwiązania równania różniczkowego

where $f(x)$, $g(x)$, $h(x)$ are the given functions of class $L^2[0,1]$, y_0 and y_1 are the given real numbers and $y(x)$ is the unknown function.

To find the approximation of function $y(x)$ we proceed as in case of the first order linear differential equation. We substitute into equation (37) the following functions

$$\begin{aligned} y(x) &= C^T\Psi(x), \\ y'(x) &= C^T\Psi'(x) = C^T D_\Psi \Psi(x), \\ y''(x) &= C^T D_\Psi^2 \Psi(x). \end{aligned} \tag{39}$$

We obtain:

$$C^T D_\Psi^2 \Psi + f(x) C^T D_\Psi \Psi + g(x) C^T \Psi = h(x). \tag{40}$$

From (38) and (39) we have:

$$\begin{aligned} C^T\Psi(0) &= y_0, \\ C^T\Psi(1) &= y_1. \end{aligned} \tag{41}$$

To find the solution y(x) first we collocate equation (40) in points $x_i = \frac{2i-1}{2^{r+1}-2}$, for $i = 1, 2, \ldots, 2^{r+1} - 1$. The resulting equation with boundary conditions (38) generates $2^{r+1} + 1$ linear equations. Solution of these equations is vector C which can be set into (39) and gives the approximation (37).

Example 3.3

By using wavelet base $\Psi_{j,k}$, we will find the approximate solution of the following differential equation

$$y''(x) + y(x) = -\cos x\, e^x, \qquad x \in [0, 1] \tag{42}$$

with the boundary conditions

$$y(0) = 0$$

and

$$y(1) = 0.$$

Diagram of $f(x)$ and its approximation are practically the same even for first approximation level (Figure 13) which confirms the good precision of approximation. Error analysis at the successive levels gives opportunity to compare the efficiency of approximation. Approximation errors for levels: $r = 1, 2, 3, 4$, are illustrated in Figure 14. Basing on presented diagrams we can say that with the increase of approximation level the approximation error decreases.

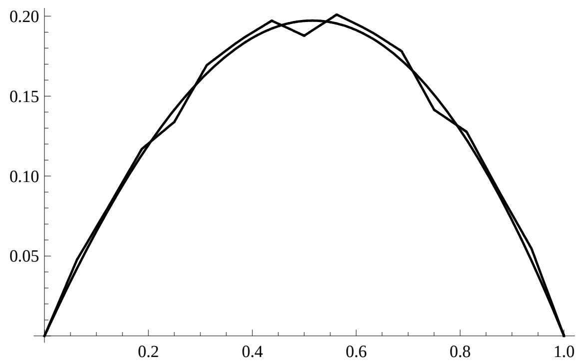

Fig. 13. Comparison of the exact solution of differential equation and its approximation for $r = 1$

Rys. 13. Porównanie przebiegu dokładnego rozwiązania równania różniczkowego i jego aproksymacji dla $r = 1$

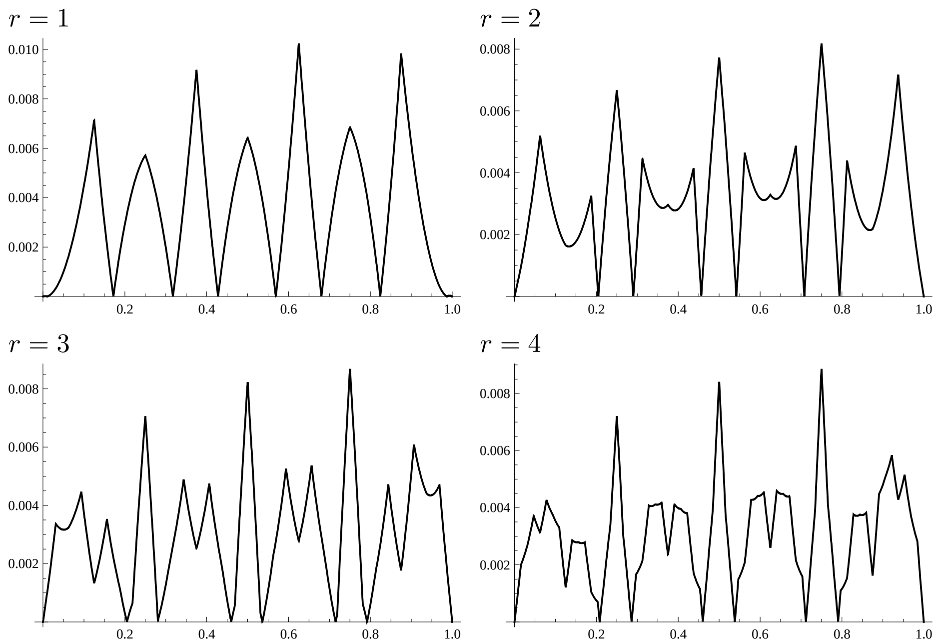

Fig. 14. Error of approximate solution of the differential equation

Rys. 14. Błąd aproksymacji rozwiązania równania różniczkowego

4. Conclusions

This paper presents the comparison of Haar's wavelet and linear B-spline wavelet with regard to the efficiency in function approximations and in approximate solutions of the first and second order differential equations. Presented examples prove that linear B-spline wavelet gives better results in functions approximation and in finding the approximate solutions of differential equations than Haar wavelet. Approximation errors calculated for each level of B-spline wavelet are much smaller than for Haar wavelet. Moreover, the examples show that errors for the Haar wavelet approximation counted for particular levels of approximation decrease much faster than approximation errors generated by using the B-spline wavelet.

References

1. Boczar T.: *Application of signal processing elements for the characteristics of acoustic emission pulses generated by partial discharges.* Phys. Chem. Solid State **8** (2007), 610–616.
2. Grzymkowski R., Zielonka A.: *Application of the wavelets theory in the boundary problems.* WPKJS, Gliwice 2004 (in Polish).
3. Jiawei X., Xuefeng Ch., Zhengjia H., Yinghong Zh.: *A new wavelet-based thin plate element using B-spline wavelet on the interval.* Comput. Mech. **41** (2008), 243–255.
4. Katunin A., Korczak A.: *The possibility of application of B-spline family wavelets in diagnostic signal processing.* Acta Mech. Autom. **3** (2009), 43–48.
5. Lakestani M., Razzaghi M., Dehghan M.: *Semiorthogonal spline wavelets approximation for Fredholm integro-differential equations.* Math. Probl. Eng. **2006** (2006), 1–12.
6. Samavati F., Mahdavi-Amiri N.: *A filtered B-spline models of scanned digital images.* J. Sci. **10** (2000), 258–264.
7. Ueda M., Lodha S.: *Wavelets: an elementary introduction and examples.* University of California at Santa Cruz, Santa Cruz 1995.

Omówienie

W niniejszym artykule dokonano porównania falki Haara i falki B-splajnowej liniowej pod względem efektywności aproksymacji funkcji oraz rozwiązań równań różniczkowych rzędów pierwszego i drugiego. Falka Haara jest „najstarszą” falką, zdefiniowaną już w 1910 roku, dobrze poznaną i obecnie powszechnie stosowaną w wielu dziedzinach. Falka Haara należy do grupy falek ortogonalnych i ze względu na jej prostą postać funkcyjną jej użycie nie wymaga skomplikowanych algorytmów. Natomiast falka B-splajnowa liniowa – nieco bardziej skomplikowana – jest falką biortogonalną, co daje szersze możliwości zastosowania jej w wielu dziedzinach nauki. Analizując przykłady zawarte w tym artykule można zauważyć, że falka B-splajnowa liniowa daje dużo lepsze rezultaty w aproksymacji rozpatrywanych funkcji oraz rozwiązań równań różniczkowych. Na każdym z poziomów rozdzielczości błędy aproksymacji, liczone dla falki B-splajnowej liniowej, były mniejsze, aniżeli liczone dla falki Haara. W wybranych przykładach można również zaobserwować, że w przypadku falki Haara błąd liczony na poszczególnych poziomach aproksymacji spada dużo szybciej niż w przypadku falki B-splajnowej. Należy podkreślić, że teoria falek B-splajnowych jest teorią stosunkowo nową, intensywnie rozwijaną przez liczne ośrodki naukowe na całym świecie. Obecnie ze względu na swoje ciekawe własności i zastosowania jest tematem wielu prac naukowych.