ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 2011
Seria: MATEMATYKA STOSOWANA z. 1 Nr kol. 1854

Edyta HETMANIOK, Danuta JAMA, Damian SŁOTA,
Adam ZIELONKA
Institute of Mathematics
Silesian University of Technology

APPLICATION OF THE HARMONY SEARCH ALGORITHM IN SOLVING THE INVERSE HEAT CONDUCTION PROBLEM

Summary. In this paper the inverse heat conduction problem with boundary condition of the third kind is solved by applying the recently invented Harmony Search algorithm belonging to the group of optimization algorithms inspired by the natural behaviors or processes. In this case the applied algorithm imitates the process of searching for the harmony in jazz music composition.

ZASTOSOWANIE ALGORYTMU „HARMONY SEARCH" DO ROZWIĄZANIA ODWROTNEGO ZAGADNIENIA PRZEWODNICTWA CIEPŁA

Streszczenie. Celem niniejszego artykułu jest rozwiązanie odwrotnego zagadnienia przewodnictwa ciepła z warunkiem brzegowym trzeciego rodzaju przy użyciu niedawno zaproponowanego algorytmu „Harmony Search" (poszukiwania harmonii). Zastosowany algorytm należy do grupy algorytmów optymalizacyjnych inspirowanych zachowaniami bądź procesami zachodzącymi w rzeczywistym świecie, w szczególności imituje proces poszukiwania harmonii dźwięków podczas improwizacji jazzowej.

2011 Mathematics Subject Classification: 80A20, 80A23.
Wpłynęło do Redakcji (received): 30.06.2011 r.

1. Introduction

Harmony Search algorithm belongs to the group of optimization algorithms imitating behaviors from the real world. Algorithms of that kind are for example the genetic algorithms, immune algorithms, Ant Colony Optimization algorithm or Artificial Bee Colony algorithm inspired by the techniques of searching for the food by the swarms of insects and methods of communication between particular individuals of the swarm. Harmony Search (HS) algorithm, proposed by Zong Woo Geem [6, 7], bases on the process of searching for the harmony of sounds by the musicians taking part in the act of jazz improvisation. All the mentioned algorithms are the heuristic algorithms which means that the solution received in result of applying the algorithm is the best, but in this particular moment. Another running of algorithm can give another solution - slightly better of slightly worse. However, this fact does not diminish the effectiveness of those algorithms, usually simple in application, rapid in action and not requiring to satisfy any specific assumptions about the solved problem.

Harmony Search algorithm has found an application in many fields of computer science and engineering. To the group of problems solved with the aid of this algorithm belong, for instance, visual tracking [5], tour planning [8], vehicle routing [9], water network design [10], soil stability analysis [4] and others.

In this paper we propose to use the HS algorithm in solving the inverse heat conduction, it means a heat conduction problem with incomplete mathematical description in which the distribution of temperature and some of the boundary conditions must be reconstructed [1,2]. This complicated problem in most of cases is impossible to solve analytically, therefore the approximate methods need to be applied. From among methods useful for solving the inverse heat conduction problem the following can be proposed: Monte Carlo method [11], Green function method [3], mollification method [14], methods based on the wavelets theory [15] or on the genetic algorithms [16,17]. Furthermore, in papers [12,13] the ACO and ABC algorithms, respectively, have been applied for minimizing the functional, being an important element in the procedure of solving the inverse heat conduction problem with reconstructing the boundary condition of the second kind. In the current paper we consider the inverse heat conduction problem with boundary condition of the third kind.

2. Harmony Search algorithm

Idea of the Harmony Search algorithm is based on the similarity between the process of jazz improvisation and the problem of optimizing the function. Jazz improvisation consists in finding the best state of harmony, similarly as the optimization algorithm consists in finding the argument realizing minimum of the function. When one of the musicians plays a note, the others must remember its sound and select the other notes such that a harmonic music can be composed. Successively, the musicians remember the notes played before, add the next notes and improve them such that the most beautiful music will arise from the chaos. Described process can be defined as the optimization of jazz composition.

Referring to the problem of the function optimization we can consider the arguments of function as the notes and the values for these arguments as the tones of instruments caused by these notes. And similarly like the musicians are searching for the combination of notes giving the best harmony of the music, we are seeking the argument in which minimum of the function is taken.

We begin the algorithm by selecting the random set of notes (arguments) and ordering them with regard to the values of minimized function in the harmony memory vector (HM). In the next step we try to improve randomly the harmony given by the combination of selected notes. We can choose the note already collected in the harmony memory vector – we can test such note one more time or we can change it slightly in hope of improving the general harmony. We can also try to find the completely new notes. Each note is put in the right order in the HM vector. After the assumed number of iterations the first element of HM vector is taken as the solution.

In details, the algorithm is of the following form.

1. Initial data:

    minimized function $f(x_1, ..., x_n)$;

    range of the variables $a_i \leqslant x_i \leqslant b_i, \quad i = 1, ..., n$;

    size of the harmony memory vector $HMS \quad (1 - 100)$;

    harmony memory considering rate coefficient $HMCR \quad (0.7 - 0.99)$;

    pitch adjusting rate coefficient $PAR \quad (0.1 - 0.5)$;

    number of iterations IT.

2. Preparation of the harmony memory vector HM – we randomly select HMS number of vectors $(x_1, ..., x_n)$ and we order them in vector HM according to the increasing values $f(x_1, ..., x_n)$:

$$HM = \begin{bmatrix} x_1^1, ..., x_n^1 & \mid & f(\mathbf{x}^1) \\ \vdots & \mid & \vdots \\ x_1^{HMS}, ..., x_n^{HMS} & \mid & f(\mathbf{x}^{HMS}) \end{bmatrix}.$$

3. Selection of the new harmony $\mathbf{x}' = (x'_1, ..., x'_n)$.

   For each $i = 1, ..., n$ the element x'_i is selected:
   - with the probability equal to $HMCR$, from among numbers x_i collected in the harmony memory vector HM;
   - with the probability equal to $1 - HMCR$, randomly from the assumed range $a_i \leqslant x_i \leqslant b_i$.

   If in the previous step the element x'_i is selected from the harmony memory vector HM then:
   - with the probability equal to PAR, we modify the element x'_i in the following way: $x'_i \rightarrow x'_i + \alpha$ (we regulate the sound of the note), for $\alpha = bw \cdot u$, where bw denotes the bandwidth – part of range of the variables and u is the randomly selected number from interval $\in [-1, 1]$;
   - with the probability equal to $1 - PAR$ we do nothing.

4. If $f(\mathbf{x}') < f(\mathbf{x}^{HMS})$ then we put the element $\mathbf{x}'$ into the harmony memory vector HM in place of the element $\mathbf{x}^{HMS}$ and we order vector HM according to the increasing values of the minimized function.

5. Steps 2–4 are repeated IT number of times. The first element of vector HM defines the solution.

3. Formulation of the problem

The considered inverse heat conduction problem is described by the equation of the form

$$c\,\rho \frac{\partial u}{\partial t}(x,t) = \lambda \frac{\partial^2 u}{\partial x^2}(x,t), \qquad x \in [0,d], \quad t \in [0,T] \tag{1}$$

with the following initial and boundary conditions

$$u(x,0) = u_0, \qquad x \in [0,d], \tag{2}$$

$$\frac{\partial u}{\partial x}(0,t) = 0, \qquad t \in [0,T], \tag{3}$$

where c is the specific heat, ρ denotes the mass density, λ is the thermal conductivity and u, t and x refer to the temperature, time and spatial location. On the boundary for $x = d$ the boundary condition of the third kind is defined

$$-\lambda\frac{\partial u}{\partial x}(d,t) = \alpha\left(u(d,t) + u_\infty\right), \qquad t \in [0,T], \tag{4}$$

where u_∞ describes the temperature of environment and α denotes the unknown heat transfer coefficient. Determination of the value of coefficient α is one of the goal of the formulated procedure. Another goal is to reconstruct the distribution of temperature $u(x,t)$ in the considered region. By setting the value α of heat transfer coefficient as given, the problem, defined by equations (1)-(4), turns into the direct problem, solving of which determines the values of temperature $u(x_i,t_j)$ in the nodes of the mesh.

In the approach presented in this paper we propose to solve the direct heat conduction problem, described by equations (1)-(4), by taking the value of heat transfer coefficient as an unknown parameter α. Solution $u(x_i,t_j)$ received in this way depends on the parameter α. Next, we determine the value of α by minimizing the following functional:

$$P(\alpha) = \sqrt{\sum_{j=1}^{m}\big(u(d,t_j) - \tilde{u}(d,t_j)\big)^2}, \tag{5}$$

representing the differences between the obtained results u and given values $\tilde{u}$ on the boundary for $x = d$, where the boundary condition is reconstructed. For minimizing the functional (5) we use the HS algorithm.

4. Numerical example

Effectiveness of the proposed approach will be illustrated by an example in which $c = 1000\,[\mathrm{J/(kg\cdot\ K)}]$, $\rho = 2679\,[\mathrm{kg/m^3}]$, $\lambda = 240\ \ [\mathrm{W/(m\cdot\ K)}]$, $T = 1000\,\mathrm{s}$, $d = 1\,\mathrm{m}$, $u_0 = 1013\,\mathrm{K}$ and $u_\infty = 298\,\mathrm{K}$. We know the exact value of the sought heat transfer coefficient which is the following: $\alpha = 28\,[\mathrm{W/(m^2\cdot\ K)}]$. For constructing the functional (5) we use the exact values of temperature, calculated for the known exact value α, and values noised by the random error of 1%. Measurement points are located on the boundary for $x = 1$, where the boundary condition should be reconstructed, with the step equal to 1 s.

Harmony Search algorithm is run for the following values of parameters:

$$HMS = 10, 15, 20,$$
$$HMCR = 0.85,$$
$$PAR = 0.3,$$
$$IT = 20 - 200.$$

Elements of the initial harmony memory vector are randomly selected from the range $[0, 500]$ and the bandwidth parameter bw corresponds with 10% of the range of variables. The approximate values of reconstructed parameter is received by running the algorithm 30 times and by averaging the obtained results.

We investigated efficiency of the Harmony Search algorithm in regard to length of harmony memory vector (parameter HMS) and number of iterations (parameter IT). Figure 1 presents the reconstructed values of heat transfer coefficient α depending on the number of iterations received for 10 elements in harmony memory vector. We can see that satisfying results, near the exact value of 28, are obtained for about 75 iterations.

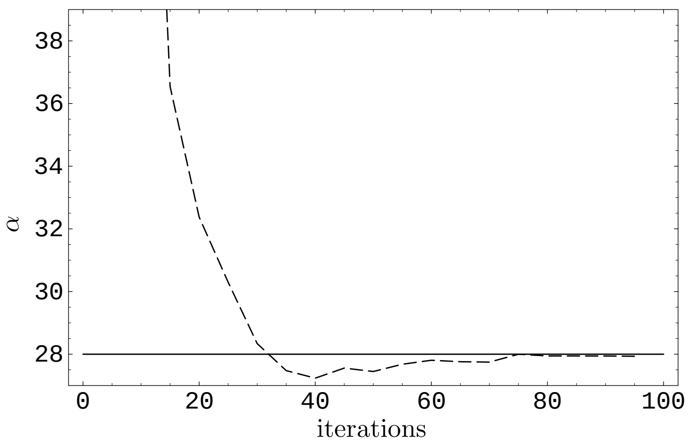

Fig. 1. Reconstructed values of parameter α (dashed line) depending on the number of iterations calculated for 10 elements in HM vector

Rys. 1. Odtworzone wartości parametru α (linia przerywana) w zależności od liczby iteracji, obliczone dla 10 elementów wektora HM

Similar relations are displayed in Figures 2 and 3, for 15 and 20 elements in HM vector, respectively. Increasing number of elements in HM vector causes the smaller number of iterations needed for receiving the satisfying results. In case of the length of HM vector equal to 15, we start to receive good reconstruction of α, on the level near by 28, for about 40 iterations, however for 20 elements in HM vector the results for 40 iterations are much better.

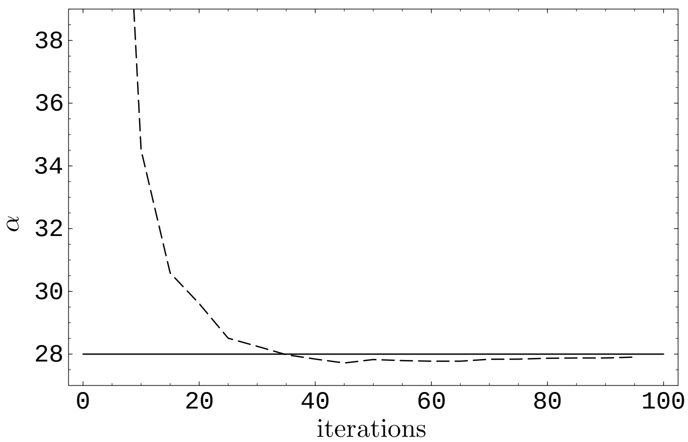

Fig. 2. Reconstructed values of parameter α (dashed line) depending on the number of iterations calculated for 15 elements in HM vector

Rys. 2. Odtworzone wartości parametru α (linia przerywana) w zależności od liczby iteracji, obliczone dla 15 elementów wektora HM

Relative errors of the parameter α reconstruction received for various numbers of elements in harmony memory vector and for various numbers of iterations are compiled in Table 1. Summarizing we may state that Harmony Search algorithm, used for solving the considered problem, is characterized by the fact that after small number of iterations the result approaches to the exact value for about 1%, however, afterwards the algorithm is convergent very slowly. From the certain moment algorithm does not enable to improve the result.

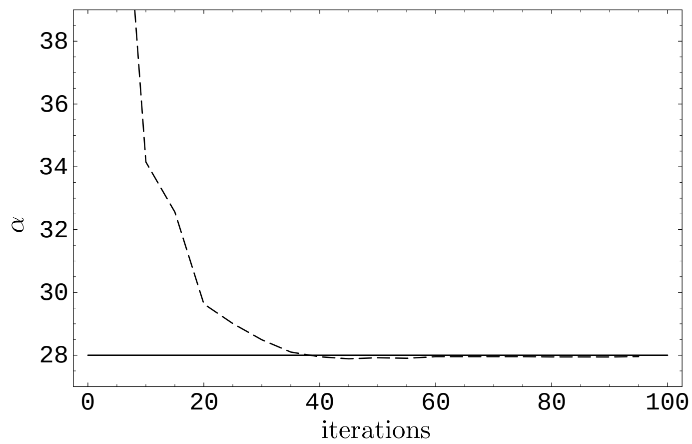

Fig. 3. Reconstructed values of parameter α (dashed line) depending on the number of iterations calculated for 20 elements in HM vector

Rys. 3. Odtworzone wartości parametru α (linia przerywana) w zależności od liczby iteracji, obliczone dla 20 elementów wektora HM

Table 1

Error in α reconstruction

IT	Δ_α [%]	Δ_α [%]	Δ_α [%]
	$HMS = 10$	$HMS = 15$	$HMS = 20$
20	15.63	5.76	1.11
30	1.23	0.91	0.85
40	2.73	0.56	0.39
50	1.58	1.01	0.37
100	0.38	0.35	0.33

5. Conclusions

In this paper we have presented the method of solving the inverse heat conduction problem with boundary condition of the third kind by using the Harmony Search algorithm as a tool of minimizing the appropriate functional being a crucial part of the proposed approach. Results received for the example, in which value of the unknown heat transfer coefficient was reconstructed, are satisfying for small number of iterations and relatively small number of elements in the harmony memory vector. Approximate result is close to the expected value with the error less than 1%, however, improvement of the precision of the received result seems to be impossible from a certain moment. Nevertheless, we may conclude that Harmony Search algorithm is useful tool in solving optimization problems and an indisputable advantage of the HS algorithm is its simplicity and universality. The only assumption needed by this algorithm is the existence of solution.

References

1. Beck J.V., Blackwell B.: *Inverse problems. Handbook of numerical heat transfer*. Wiley Intersc., New York 1988.
2. Beck J.V., Blackwell B., St.Clair C.R.: *Inverse heat conduction: ill posed problems*. Wiley Intersc., New York 1985.
3. Beck J.V., Cole K.D., Haji-Sheikh A., Litkouhi B.: *Heat conduction using Green's functions*. Hempisphere Publishing Corporation, Philadelphia 1992.
4. Cheng Y.M., Li L., Lansivaara T., Chi S.C., Sun Y.J.: *An improved Harmony Search minimization algorithm using different slip surface generation methods for slope stability analysis*. Eng. Optim. **40** (2008), 95–115.
5. Fourie J., Mills S., Green R.: *Visual tracking using the harmony search algorithm*. Image and Vision Computing New Zealand, 23rd International Conference (2008), 1–6.
6. Geem Z.W.: *Improved Harmony Search from ensemble of music players*. Lecture Notes in Artificial Intelligence **4251** (2006), 86–93.
7. Geem Z.W., Kim J.H., Loganathan G.V.: *A new heuristic optimization algorithm: Harmony Search*. Simulation **76** (2001), 60–68.
8. Geem Z.W., Tseng C.L., Park Y.: *Harmony Search for generalized orienteering problem: best touring in China*. Lecture Notes in Comput. Sci. **3612** (2005), 742–750.
9. Geem Z.W., Lee K.S., Park Y.: *Application of Harmony Search to vehicle routing*. American Journal of Applied Sciences **2** (2005), 1552–1557.
10. Geem Z.W.: *Optimal cost design of water distribution networks using Harmony Search*. Eng. Optim. **38** (2006), 259–280.
11. Haji-Sheikh A., Buckingham F.P.: *Multidimensional inverse heat conduction using the Monte Carlo method*. Trans. of ASME, Journal of Heat Transfer **115** (1993), 26–33.

12. Hetmaniok E., Słota D., Zielonka A.: *Solution of the inverse heat conduction problem by using the ABC algorithm.* Lecture Notes in Comput. Sci. **6086** (2010), 659–668.
13. Hetmaniok E., Zielonka A.: *Solving the inverse heat conduction problem by using the ant colony optimization algorithm.* CMM-2009, University of Zielona Góra Press (2009), 205–206.
14. Mourio D.A.: *The mollification method and the numerical solution of ill-posed problems.* John Wiley and Sons, New York 1993.
15. Qiu C.Y., Fu C.L., Zhu Y.B.: *Wavelets and relularization of the sideways heat equation.* Comput. Math. Appl. **46** (2003), 821–829.
16. Słota D.: *Solving the inverse Stefan design problem using genetic algorithm.* Inverse Probl. Sci. Eng. **16** (2008), 829–846.
17. Słota D.: *Restoring boundary conditions in the solidification of pure metals.* Comput. & Structures **89** (2011), 48–54.

Omówienie

Celem niniejszego artykułu było przedstawienie propozycji metody rozwiązywania odwrotnego zagadnienia przewodnictwa ciepła z odtwarzanym warunkiem trzeciego rodzaju. Nowatorskim elementem proponowanej metody jest zastosowanie algorytmu poszukiwania harmonii (Harmony Search algorithm) do minimalizacji pewnego funkcjonału, odgrywającego istotną rolę w procedurze. Algorytm poszukiwania harmonii jest interesującym przykładem algorytmu optymalizacyjnego, opartego na naśladowaniu zachowań i zjawisk zachodzących w otaczającej nas rzeczywistości. Wykorzystany algorytm zainspirowany został podobieństwem między procesem budowania harmonii dźwięków podczas improwizacji jazzowej a poszukiwaniem argumentu, realizującego minimum funkcji. Zastosowanie algorytmu HS dało zadowalające rezultaty dla niewielkiej liczby iteracji oraz stosunkowo niewielkiej liczby elementów wektora pamięci harmonii, jednak poprawienie precyzji wyniku od pewnego momentu okazało się niemożliwe. Niemniej jednak algorytm „Harmony Search” można uznać za przydatne narzędzie optymalizacyjne, a jego niewątpliwymi zaletami są prostota i uniwersalność. Jedynym warunkiem, koniecznym do zastosowania algorytmu, jest istnienie rozwiązania optymalizowanego problemu.