On arrangement of subgroups in groups and related topics

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Subgroups of a group allow a wide variety of arrangements. Among them we can mention their pair wise dispositions and their dispositions relatively to the group.

Investigation of groups satisfying certain related to the subgroup arrangement conditions enabled algebraists to introduce and describe many important classes of groups. The roots of such investigations lie in the works by P. Hall, R. Carter, J. Rose, and Z.I. Borevich.

Let G be a group and G_0 its subgroup. *A* subgroup H is called *intermediate* to G_0 if

$G_0 \leq H \leq G.$

Suppose G is a group and G_0 a subgroup of G. A system $\{G_\alpha, N_G(G_\alpha) \mid \alpha \in I\}$ of intermediate to G_0 subgroups and their normalizers in G is called *a fan for* G_0 if for each intermediate subgroup H there exists a unique index $\alpha \in I$ such that $G_\alpha \leq H \leq N_\alpha$ (Borevich, 1980).

The subgroups G_α are called *the basis subgroups of this fan*.

If there exists a fan for G_0, then G_0 is called *a fan subgroup* of G.

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Borevich, Z.I., Vavilov, N.A. Arrangement of subgroups in the general linear group over a commutative ring. Trudy Mat. Inst. Steklov.(LOMI)-165 (1984), 24-42

Gawron, P.V., Kolotilina, N. Yu. On subgroups of T-groups. Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) -103 (1980), 62-65].

N.F. Kuzennii and I.Ya. Subbotin in 1990 described the groups *G* each subgroup *D* of which has a fan with a basis that is a subset of the set *{D,G},* the groups *G* containing a fixed subgroup $M(G) \neq G$ such that each non-normal subgroup *D* of *G* has a fan with the basis *{D,M(G)}*, and the groups each non-invariant subgroup of which has a fan with the basis consisting of all intermediate subgroups.

Consider some other important fan subgroups.

A subgroup *H* of a group *G* is said to be *abnormal* in *G* if $g \in \langle H,H^g \rangle$ for each element $g \in G$. [P. Hall 1937]

The term *an abnormal subgroup* is due to R. Carter (1961).

A subgroup *H* of a group *G* is said to be *pronormal* in *G* if for every $g \in G$ the subgroups *H* and H^g are conjugate in the subgroup $\langle H,H^g \rangle$

[P. Hall].

A *contranormal subgroup* is a subgroup *H* of a group *G* such that $H^G = G$ [J. Rose (1968)].

A subgroup *D* is said to be *polynormal* in *G* if it is a fan subgroup and all intermediate subgroups *H* such that $D^H=H$ form the system of the basis subgroups for its fan, i.e. *D* is polynormal in *G* if for every H, $D \leq H$, *D* is contranormal in D^H.

D___contranormal ___D^H____normal____H

[Z.I. Borevich and O.N. Macedonska, 1980).

These definitions **have no limitation of finiteness.**

All abnormal, pronormal and some other important types of fan subgroups are polynormal.

If *D* is a pronormal subgroup in *G*, then $N_G(D)$ is abnormal in *G*.

D*_normal_____ $N_G(D)$__abnormal_____*G

In 1980th, **Z.I. Borevich** introduced **polynormal, paranormal, weakly pronormal, and weakly abnormal subgroups**.

A subgroup H of a group G is said to be *weakly pronormal* in G if for any two intermediate subgroup K, L for H such that K is normal in L the following inclusion holds $L \leq N_G(H)K$ (in this case, we will say also that H has *the Frattini property*).

A subgroup H is called *weakly abnormal* in G if $x \in H^{\langle x \rangle}$ for each element $x \in G$.

1. Abnormal subgroups and their generalizations

A subgroup of a group is simultaneously normal and abnormal only if it coincides with the group. Maximal non-normal subgroups are trivial examples of abnormal subgroups. More interesting is a well-known J. Tits example: a subgroup ***T****(n, K)* of all triangular matrices is abnormal in a general linear group *GL(n, F)* over a field *F*. Every Carter subgroup (that is a nilpotent self-normalizing subgroup) of finite soluble group is abnormal [R.W. Carter, **1961**].

Theorem [Carter, 1961]. ***Let G be a group and H a subgroup of G. Then H is abnormal in G if and only if the following two conditions hold:***

(i) If K is an intermediate subgroup for H, then K is self-normalizing.

(ii) If K, L are two intermediate subgroup for H such that $L = x^{-1} K x$***, then*** $K = L$***.***

Theorem [Ba & Borevich, 1988]. ***Let G be a group and H be a subgroup of G. Then H is weakly abnormal in G if and only if every intermediate subgroup for H is self-normalizing.***

In the case of soluble groups, the condition (ii) could be omitted. For finite soluble groups this fact is mentioned in the book of B. Huppert [p. 733, Theorem 11.17].

A group G is called *an Ñ-group* if G satisfies the following condition:

If M, L are subgroup of G such that M is maximal in L, then M is normal in L.

We observe that the property “to be an Ñ-group” is local [A.G. Kurosh and S.N. Chernikov, 1947]. In particular, every locally nilpotent group is an Ñ-group, but converse is not true [J. Wilson, 1977].

Theorem (Kurdachenko & Subbotin, 2010). ***Let G be a hyper-Ñ-group and H be a subgroup of G. Then H is abnormal in G if and only if every intermediate subgroup for H is self-normalizing.***

Corollary [Kurdachenko & Subbotin, 2005]. ***Let G be a radical group and H be a subgroup of G. Then H is abnormal in G if and only if every intermediate subgroup for H is self-normalizing.***

Corollary [De Giovanni & Vinchenzi, 2001]. ***Let G be a hyperabelian group and H be a subgroup of G. Then H is abnormal in G if and only if every intermediate subgroup for H is self-normalizing.***

Corollary. ***Let G be a soluble group and H be a subgroup of G. Then H is abnormal in G if and only if every intermediate subgroup for H is self-normalizing.***

If *H* is weakly abnormal (respectively, abnormal) in *G*, and *K* is an intermediate subgroup for *H*, then *H* is weakly abnormal (respectively, abnormal) in *K*, and *K* is weakly abnormal (respectively, abnormal) in *G*.

In $G =$ ***Sym***(4) consider
Sym$(3)=H = \langle (1\ 2), (1\ 2\ 3) \rangle$ and $D = \langle (1\ 2) \rangle$.
The subgroup *D* is maximal in *H*, but not normal, so that *D* is abnormal in *H*. The subgroup *H* is abnormal in *G*. However, *D* is not abnormal (and not weakly abnormal) in *G*
(not all its intermediate subgroups are self normalizing; even $N_G(D)= \langle (1\ 2), (34) \rangle \neq D$) .

Consequently, the property "*to be an abnormal subgroup*" ("*to be weakly abnormal*") is not transitive.

However,

Proposition (P. Hall). ***Let G be a group and H be a normal subgroup of G. If a subgroup D is abnormal in DH and DH is abnormal in G, then D is abnormal in G.***

Theorem (Kurdachenko, Subbotin, 2005). ***Let G be a group and H be a normal subgroup of G. Suppose that G/H has no proper abnormal subgroups and H satisfies the normalizer condition. Then abnormality is transitive in G.***

Abnormal subgroups are contranormal. However not every contranormal subgroup is abnormal.

Observe that in a soluble group, an abnormal subgroup *R* is exactly the subgroup that is contranormal in all subgroups containing *R* [De Falko, Kurdachenko, Subbotin, 1998].

A subgroup *H* of a group *G* is said to be *nearly abnormal***,** if *H* is contranormal in *K* for every subgroup *K* containing *H*. This condition *to be nearly abnormal* is an amplification of the transitivity of abnormality.

Theorem (Kurdachenko, Subbotin, 2010). ***Let G be a hyper-Ñ-group and H be a subgroup of G. Then H is nearly abnormal in G if and only if H is abnormal in G.***

We call ***the U-normal subgroups*** (from the *"union"* and "*U-turn*") the union of normal and abnormal subgroups. The finite groups with only *U*-normal subgroups have been considered by Fattahi in 1974. Locally soluble (in the periodic case, locally graded) infinite groups with only *U*-subgroups have been studied by Kurdachenko and Subbotin in 2002. They described the groups with all *U*-normal subgroups and the groups with transitivity of *U*-normality. Observe that a union of any two *U*-normal subgroups is *U*-normal while the similar assertion for intersections is obviously false.

The next natural question is regarding the structure of the *groups whose U-normal subgroups form a lattice*. These groups are denoted as *#U-groups* [Kurdachenko and Subbotin, 2007]. The description of some soluble groups having *#U*-property was obtained by Kurdachenko and Subbotin in 2007.

2. Pronormal subgroups and their generalizations

Let G be a group. A subgroup H is called *weakly pronormal in G* if the subgroups H and H^x are conjugatein $H^{<x>}$ for each element $x \in G$ [Ba & Borevich, 1988].

The inclusion $< H, H^x > \leq H^{<x>}$ shows that **every pronormal subgroup is weakly pronormal**. The converse is not true.

Let G be a group and H be a subgroup of G. We say that H has the *Frattini property* if for every subgroups K, L such that $H \leq K$ and K is normal in L we have $L = N_L(H) K$.

Theorem [Ba & Borevich, 1988]. ***Let G be a group and H be a subgroup of G. Then H is weakly pronormal in G if and only if H has the Frattini property.***

Corollary. ***Let G be a group and H be a pronormal subgroup of G. Then H has the Frattini property.***

Corollary. ***Let G be a group and H be a weakly pronormal subgroup of G. Then H is weakly abnormal in G if and only if $H = N_G(H)$.***

Theorem [Ba & Borevich, 1988]. ***Let G be a group and H be a subgroup of G. Then H is pronormal in G if and only if the following conditions hold:***

(i) H is weakly pronormal;

(ii) if L is a intermediate subgroup for H and g is an element of G such that $H \leq L^g$, then there exists an element $x \in N_G(H)$ with the property $L^x = L^g$.

Corollary. ***Let G be a group and H be a pronormal subgroup of G. Then H is abnormal in G if and only if $H = N_G(H)$.***

Proposition. ***Let G be a group and H be a subgroup of G. If H is pronormal in G, then $N_G(H)$ is abnormal in G.***

Theorem (T. A. Peng, 1971). ***Let G be a finite soluble group and D be a subgroup of G. Then D is pronormal in G if and only if D posses the Frattini property.***

Theorem (Kurdachenko, Otal, Subbotin, 2005). ***Let G be a hyper-N-group and D be a subgroup of G. Then D is pronormal in G if and only if D posses the Frattini property.***

Corollary (F. de Giovanni, G. Vincenzi, 2001). ***Let G be a hyperabelian group and D be a subgroup of G. Then D is pronormal in G if and only if D posses the Frattini property.***

Corollary. ***Let G be a soluble group and D be a subgroup of G. Then D is pronormal in G if and only if D posses the Frattini property.***

A subgroup *H* of a group *G* is called *nearly pronormal* if *$N_L(H)$* is contranormal in *L* for every subgroup *L* containing *H*.

As we can see, every pronormal subgroup is nearly pronormal, but converse is not true.

In a special unitary group of 3×3 matrices over a field $\mathbb{F}_9$ of order 9, there is a nearly pronormal, but not pronormal subgroup isomorphic to ***Sym****(4)*.

Proposition (L.A. Kurdachenko, A.A. Pypka, I.Ya. Subbotin, 2010). ***Let G be a group having ascending series whose factors are abelian. Then every nearly pronormal subgroup of G is weakly pronormal in G.***

Theorem (L.A. Kurdachenko, A.A. Pypka, I.Ya. Subbotin, 2010). ***Let G be a hyper-N-group. Then every nearly pronormal subgroup of G is pronormal in G.***

Corollary (L.A. Kurdachenko, A.A. Pypka, I.Ya. Subbotin, 2010). ***Let G be a soluble group. Then every nearly pronormal subgroup of G is pronormal in G.***

Corollary. ***Let G be a soluble group. Suppose that a subgroup H satisfies the following condition:***

If K is a subgroup containing H, then $N_K(H)$ is abnormal in K.

Then K is pronormal in G.

Corollary (Wood G.J., 1974). ***Let G be a finite soluble group. Suppose that a subgroup H satisfies the following condition:***

If K is a subgroup containing H, then $N_K(H)$ is abnormal in K.

Then K is pronormal in G.

We observe that for the generalized pronormal subgroups the class of an *Ñ*-group plays a special role.

Proposition. ***Let G be an Ñ-group and H be a nearly pronormal subgroup of G. Then H is normal in G.***

Corollary. ***Let G be a locally nilpotent group and H be a nearly pronormal subgroup of G. Then H is normal in G.***

Proposition. ***Let G be an Ñ-group and H be a weakly pronormal subgroup of G. Then H is normal in G.***

Corollary. ***Let G be a locally nilpotent group and H be a weakly pronormal subgroup of G. Then H is normal in G.***

Corollary. ***Let G be an Ñ-group and H be a pronormal subgroup of G. Then H is normal in G.***

Corollary (N.F. Kuzennyi, I.Ya Subbotin, 1988). ***Let G be a locally nilpotent group and H be a pronormal subgroup of G. Then H is normal in G.***

T.A. Peng, 1969 has considered finite groups whose all subgroups are pronormal**. It was proved to be the groups in which the relation ”to be a normal subgroup” is transitive.**

A group G is said to be *T-group* if every subnormal subgroup of G is normal. A group G is said to be a $\overline{T}$*-group*, if every subgroup of G is a T-group.

W. Gaschütz, 1957, proved that **every finite soluble T-group is a $\overline{T}$-group.**

Theorem (D.J.S. Robinson, 1964). ***Let G be a locally*** **soluble $\overline{T}$-group.**

(i) If G is not periodic, then G is abelian.

(ii) If G is periodic and L is the locally nilpotent residual of G, then G satisfies the following conditions:

(a) G/L is a Dedekind group;

(b) $\Pi(L) \cap \Pi(G/L) = \varnothing$;

(c) $2 \notin \Pi(L)$;

(d) every subgroup of L is G-invariant.

In particular, if L is non-identity, then L = [L, G].

Note that in general case, **the locally nilpotent residual does not have to be complemented.**

Theorem (T.A. Peng, 1969). ***Let G be a finite soluble group. Then every subgroup of G is pronormal if and only if G is a T-group.***

We recall that a group is called ***locally graded***, if every its finitely generated non-identity subgroup has a proper subgroup of finite index.

Theorem (Kuzennyi and Subbotin, 1987). ***Let G be a locally soluble group or a periodic locally graded group. Then the following conditions are equivalent: (i) every cyclic subgroup of G is pronormal in G;***

(ii) G is a soluble $\bar{T}$-group.

Theorem (Kuzennyi and Subbotin, 1987). ***Let G be a group whose subgroups are pronormal, and L be a locally nilpotent residual of G.***

(i) If G is periodic and locally graded, then G is a soluble $\bar{T}$-group, in which L is a complement to every Sylow $\Pi(G/L)$-subgroup.

(ii) If G is not periodic and locally soluble, then G is abelian.

Conversely, if G has this structure, then every subgroup of G is pronormal in G.

N.F. Kuzennyi and I.Ya. Subbotin also described the **locally graded periodic groups in which all primary subgroups are pronormal (1989), infinite locally soluble groups in which all infinite subgroups are pronormal (1988), groups in which all abelian subgroups are pronormal (1992).**

Theorem (L.A.Kurdachenko, A. Russo, G. Vincenzi, 2007). ***Let G be a locally radical group.***

(i) If every cyclic subgroup of G is nearly pronormal, then G is $\bar{T}$***-group.***

(ii) If every subgroup of G is nearly pronormal, then every subgroup of G is pronormal in G.

J. Rose (1965) has introduced a *balanced chain* connecting a subgroup *H* to a group *G*, that is, a chain of subgroups

$$H = H_0 \leq H_1 \leq . . . \leq H_{n-1} \leq H_n = G$$

such that for each j, $0 \leq j \leq n\text{-}1$, either H_j is normal in H_{j+1}, or H_j is abnormal in H_{j+1}. The number *n* is the length of this chain. In finite groups, every subgroup can be connected to the group by some balanced chain.

If the lengths of these chains are 1, then every subgroup is either normal or abnormal in a group.

Such finite groups were studied by Fattahi in 1974. Infinite groups of this kind and some their generalizations were studied by Subbotin in 1992, and De Falko, Kurdachenko and Subbotin in 1998. In this setting, **the groups whose subgroups are either abnormal or subnormal** have been considered.

L.A. Kurdachenko and H. Smith, **2005** considered the **groups whose subgroups are either self-normalizing or subnormal**.

Observe that in the groups in which a normalizer of any subgroup is abnormal, and in the groups in which every subgroup is abnormal in its normal closure the mentioned lengths are at most *2*.

If *G* is a soluble $\overline{T}$-group, then every subgroup of *G* is abnormal in its normal closure. For any pronormal subgroup *H* of a group *G*, the normalizer $N_G(H)$ is an abnormal subgroup of *G*. So a subgroup having abnormal normalizers is a generalization of a pronormal subgroup.

Theorem (L.A. Kurdachenko, A. Russo, I.Ya. Subbotin, G. Vincenzi, 2008).

(i) Let G be a radical group. Then G is a $\bar{T}$-group if and only if every cyclic subgroup of G is abnormal in its normal closure.

(ii) Let G be a periodic soluble group. Then G is a $\bar{T}$-group if and only if its locally nilpotent residual L is abelian and the normalizer of each cyclic subgroup of G is abnormal in G.

Theorem (Kurdachenko, Russo, Subbotin, Vincenzi, 2008). ***Let G be a periodic soluble group. Then every subgroup of G is pronormal if and only if its locally nilpotent residual L is abelian and the normalizer of every subgroup of G is abnormal in G.***

In the non-periodic case, there exist non-periodic non-abelian groups in which normalizers of all subgroups are abnormal. On the other hand, the non-periodic locally soluble groups in which all subgroups are pronormal are abelian [Kuzennyi, Subbotin, 1987].

Theorem (Kurdachenko, Russo, Subbotin, Vincenzi, 2008) ***Let G be a non-periodic group with an abelian locally nilpotent residual L. If a normalizer of every cyclic subgroup is abnormal and for each prime p $\in \Pi$(L) the Sylow p-subgroup of L is bounded, then G is abelian.***

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