Ideals in the ring $T(\infty, \mathbb{F})$

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Infinite matrices : definition

$M(\infty, K)$ - $\mathbb{N} \times \mathbb{N}$ matrices over K:

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$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ a_{21} & a_{22} & a_{23} & \cdots \\ a_{31} & a_{32} & a_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

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$C = AB$
if infinite sum $\sum_{k=1}^{\infty} a_{ik} b_{kj}$ contains finite number of nonzero elements, then this sum is noted as c_{ij}

Rings of matrices

$T(\infty, K)$ describes a ring of infinite upper triangular matrices with unity.

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- I is both sided ideal (ideal) if is simultaneously left and right ideal

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$A_{i*} = a$ describes that every element from i-th row is equal a, the same is for columns.

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Thank you for attention.

References

- A.K. Suszkiewicz *On an infinite algebra of triangular matrices.(Russian)* Har'kov. Gos. Univ. Uć. Zap. 34 = Zap. Mat. Otd. Fiz.-Mat. Fak. i Har'kov. Mat. Obść.(4) **22** (1950), 77–93.
- W. Hołubowski, M. Maciaszczyk, S. Żurek *Note on Suškevič's problem on zero divisors* Comm. in Algebra **vol. 45, no. 8** (2017), 3274–3277.