GROUPS AND THEIR ACTIONS

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ABSTRACTS

Associativity degree in finite Moufang loops

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The *commutativity degree*, $Pr(G)$, of a finite group G (i.e. the probability that two (randomly chosen) elements of G commute with respect to its operation)) has been studied well by many authors, recently. It is well-known that the best upper bound for $Pr(G)$ is $\frac{5}{8}$ for a finite non–abelian group G.

The speaker of this talk has defined the same concept for a finite non–commutative *Moufang loop* M and tried to give a best upper bound for $Pr(M)$. He has proved that for a well-known class of finite Moufang loops, named *Chein loops*, and its modifications, this best upper bound is $\frac{23}{32}$ and conjectured is that *for any finite Moufang loop* M, $Pr(M) \leq \frac{23}{32}$.

In this talk, we will define a similar and new notion about the associativity for a finite loop and call it associativity degree of that. So we define, the *associativity degree*, $Pas(L)$, of a finite loop L as the probability that three (randomly chosen) elements of L associate with respect to its operation.

We try to obtain a best upper bound for $Pas(M)$, where M is a finite non–associative *Moufang loop*. We will show that for the class of Chain loops, and its modifications, this best upper bound is $\frac{43}{64}$ and it is related to the commutativity degree of M, $Pr(M)$. Here is also, the conjecture is: *for any finite Moufang loop* M, $Pas(M) \leq \frac{43}{64}$. Finally, we propose some questions for associativity degree of finite Moufang loops, which similar ones have been answered for the commutativity degree of finite groups.

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Derivations in semiprime rings

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An additive map $d : R \to R$ is called *a derivation* of a ring R if the Leibnitz rule $d(ab) = d(a)b + ad(b)$ holds for all $a, b \in R$. Let

$$D = \{d : R \to R \mid d \text{ is a derivation}\}.$$

N. Jacobson (1937) has proved that D is a Lie ring.

C.R. Jordan and D.A. Jordan (1978) have proved that *if R is a prime 2-torsion free commutative ring or a Noetherian δ-prime ring, where $\delta \in D$, then $R\delta = \{r\delta \mid r \in R\}$ and $[R\delta, R\delta]$ are prime Lie rings.* A. Nowicki (1985) have extended these results. M.A. Chebotar and P.-H. Lee (2006) have shown that *if R is a reduced (i.e., without nonzero nilpotent elements) 2-torsion free commutative D-prime ring, D is a nonzero Lie subring and an R-submodule of D, $\delta \in D$, then D is a prime Lie ring.* P.-H. Lee and C.-K. Liu (2007) have establish that *if R is a 2-torsion free commutative D-prime ring, D is a Lie subring and an R-submodule of D, then any nonzero Lie ideal A of D is a prime Lie ring.* A Lie ring D is called *(Lie) semiprime* if, for any nonzero Lie ideal J of D, we have $[J, J] \neq 0$. C.-H. Liu (2006) has proved that *if R is a 2-torsion free commutative D-semiprime ring, D is a Lie subring and an R-submodule of D, then D is Lie semiprime.*

During the talk we present some new results on (non-commutative) associative rings R with semiprime Lie ring D of derivations and derivations of differentially semiprime rings.

On Cayley Graphs on Non-Abelian Groups of Order $3pq$

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Suppose p and q are prime numbers. In this paper, the connected Cayley graph of groups of order $3pq$ are investigated and all connected $\frac{1}{2}-$arc-transitive Cayley graphs of these orders will be classified.

On Torsion Subgroups in Integral Group Rings of Almost Simple Groups

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Let G be a finite group. How much does the structure of G govern the structure of the torsion part of the unit group of the integral group ring $\mathbb{Z}G$? Already in the 1930s, Graham Higman raised the question whether all finite subgroups of the normalized units of $\mathbb{Z}G$ are isomorphic to subgroups of G. Although this is not true in general, there are far reaching positive results.

This talk will report about recent results on two particular aspects: r-subgroups of the integral group ring of $\mathrm{PSL}(2, p^3)$ (for primes p and r) and cyclic subgroups of mixed order in the integral group ring of almost simple groups with socle A_n.

On adding machine

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The (binary) adding machine can be viewed, among the others, as an automorphism of the rooted 2-adic tree. We shall provide the explicit form of the normal closure of the adding machine in the group of automorphisms of the rooted 2-adic tree. To do it we use the representation of automorphisms of rooted trees defined by L.Kaloujnine in [1]. We also give some properties.

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Intersection of conjugated solvable subgroups in a symmetric group

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Assume that a finite group G acts on a set Ω. An element $x \in \Omega$ is called a *regular point* if $|xG| = |G|$, i.e. if the stabilizer of x is trivial. Define the action of the group G on Ω^k by the rule

$$g : (i_1, \ldots, i_k) \mapsto (i_1 g, \ldots, i_k g).$$

If G acts faithfully and transitively on Ω, then the minimal number k such that the set Ω^k contains a G-regular point is called the *base size* of G and is denoted by $b(G)$. For a positive integer m the number of G-regular orbits on Ω^m is denoted by $Reg(G, m)$ (this number equals 0 if $m < b(G)$). If H is a subgroup of G and G acts by the right multiplication on the set Ω of right cosets of H then G/H_G acts faithfully and transitively on the set Ω. (Here $H_G = \cap_{g \in G} H^g$.) In this case, we denote $b(G/H_G)$ and $Reg(G/H_G, m)$ by $b_H(G)$ and $Reg_H(G, m)$ respectively.

Thus $b_H(G)$ is the minimal number k such that there exist elements $x_1, \ldots, x_k \in G$ for which $H^{x_1} \cap \ldots \cap H^{x_k} = H_G$.

Consider the problem 17.41 from "Kourovka notebook" [1]:

Let H be a solvable subgroup of finite group G and G does not contain nontrivial normal solvable subgroups. Are there always exist five subgroups conjugated with H such that their intersection is trivial?

The problem is reduced to the case then G is almost simple in [2]. Specifically, it is proved that if for each almost simple group G and solvable subgroup H of G condition $Reg_H(G, 5) \geq 5$ holds then for each finite non-solvable group G and solvable subgroup H of G condition $Reg_H(G, 5) \geq 5$ holds.

We have proved the following theorem

Theorem 1. *Let H be a solvable subgroup of an almost simple group G whose socle is isomorphic to A_n, $n \geq 5$. Then $Reg_S(G, 5) \geq 5$. In particular $b_S(G) \leq 5$.*

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On the group of automorphisms of a regular ended tree

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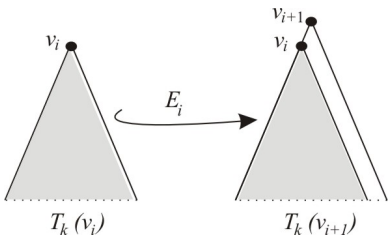
Let $T(\varepsilon)$ be an infinite regular tree with a fixed end ε, in which every vertex has valency $k+1$. The tree may be constructed as an inverse limit of a system of infinite k-adic rooted trees T_k with the following embeddings:

In the talk I will discuss the structure and some properties of the group $AutT(\varepsilon)$ of automorphisms of $T(\varepsilon)$.



Polynomial defining many units

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Let P be a polynomial in one variable and let g be a group element of order n. We say that P defines a unit in order n if $P(g)$ is a unit of the group of integral group ring of the group generated by g with integral coefficients. Marciniak and Sehgal introduced the notion of generic unit and give a characterization of them. A generic unit is a monic polynomial P with integral coefficients such that there is a positive integer D such that P defines a unit in order n for every n coprime with D. Obviously a generic unit defines units in infinitely many orders. We prove a converse of this result. More precisely we prove that if P is a polynomial with integral coefficients, non-necessarily monic, such that P defines units on infinitely many units then $P = \pm Q$, for Q a generic unit.

Groups with the property R_∞

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Let $\varphi : G \to G$ be an endomorphism of a group G. Then two elements x, y of G are said to be twisted φ-conjugate, if there exists a third element $z \in G$ such that $x = zy\varphi(z)^{-1}$. The equivalence classes are called the twisted conjugacy classes or the Reidemeister classes of φ. The Reidemeister number of φ denoted by $R(\varphi)$, is the number of those twisted conjugacy classes of φ. This number is either a positive integer or ∞ .

An infinite group G has the R_∞-property if for every automorphism φ of G the Reidemeister number of φ is infinite.

The interest in twisted conjugacy relations has its origins, in particular, in the Nielsen-Reidemeister fixed point theory, in Arthur- Selberg theory, in Algebraic Geometry, in Galois cohomology, in the theory of Linear Algebraic groups, in Representation Theory.

The problem of determining which classes of discrete infinite groups have the R_∞ property is an area of active research initiated by R. Hill and the author in 1994.

Later, it was shown by various authors that the following groups have the R_∞-property: non-elementary Gromov hyperbolic groups; relatively hyperbolic groups; Baumslag-Solitar groups $BS(m,n)$ except for $BS(1,1)$; a wide class of saturated weakly branch groups (including the Grigorchuk group and the Gupta-Sidki-Sushchansky group), Thompson's groups F and T; generalized Thompson's groups $F_{n,0}$ and their finite direct products; Houghton's groups; symplectic groups $\mathrm{Sp}(2n,\mathbb{Z})$, the mapping class groups Mod_S of a compact oriented surface S with genus g and p boundary components, $3g+p-4>0$, and the full braid groups $B_n(S)$ on $n>3$ strands of a compact surface S in the cases where S is either the compact disk D, or the sphere S^2; extensions of $SL(n,Z)$, $PSL(n,Z)$, $GL(n,Z)$, $PGL(n,Z)$, $Sp(2n,Z)$, $PSp(2n,Z)$, $n>1$, by a countable abelian group, and normal subgroups of $SL(n,Z)$, $n>2$, not contained in the center; $GL(n,K)$ and $SL(n,K)$ if $n>2$ and K is an infinite integral domain with trivial group of automorphisms, or K is an integral domain, which has a zero characteristic and for which $Aut(K)$ is periodic; irreducible lattices in a connected semisimple Lie group G with finite center and real rank at least 2; non-amenable, finitely generated residually finite groups; some metabelian groups; lamplighter groups $\mathbb{Z}_n \wr \mathbb{Z}$ if and only if $2|n$ or $3|n$; free nilpotent groups N_{rc} of rank $r=2$ and class $c\geq 9$, N_{rc} of rank $r=2$ or $r=3$ and class $c\geq 4r$, or rank $r\geq 4$ and class $c\geq 2r$, any group N_{2c} for $c\geq 4$, every free solvable group S_{2t} of rank 2 and class $t\geq 2$ (in particular the free metabelian group $M_2=S_{22}$ of rank 2), any free solvable group S_{rt} of rank $r\geq 2$ and class t big enough; some crystallographic groups. Recently it was proven that N_{rc}, $r>1$ has the R_∞-property if and only if $c\geq 2r$.

Jabara proved that if residually finite group G admits an automorphism of prime order p with finite Reidemeister number, then G is virtually nilpotent group of class bounded by a function of p.

We have described a lot of classes of non- solvable, finitely generated, residually finite groups which have the R_∞-property. All together was a motivation for the following conjecture proposed by E. Troitsky and the author

Conjecture Every infinite, residually finite, finitely generated group

either possesses the R_∞-property or is a virtually solvable group.

Let Ψ belongs to $Out(G) = Aut(G)/Inn(G)$. We consider an outer automorphism $\Psi \in Out(G)$ as a collection of ordinary automorphisms $a \in Aut(G)$. We say that two automorphisms $a, b \in \Psi$ are *similar* (or *isogredient*) if $b = \varphi_h a \varphi_h^{-1}$ for some $h \in G$, where $\varphi_h(g) = hgh^{-1}$ an inner automorphism induced by the element h. Let $S(\Psi)$ be the set of isogredience classes of automorphisms representing Ψ.

A group G is called an S_∞-*group* if for any Ψ the set $S(\Psi)$ is infinite.

In this talk we discuss the R_∞ and S_∞ properties for linear algebraic groups. The following theorem was proven by Timur Nasybullov and the author in arXiv:1506.02464:

Theorem *Let F be such an algebraically closed field of zero characteristic that the transcendence degree of F over $\mathbb{Q}$ is finite. If the reductive linear algebraic group G over the field F has a nontrivial quotient group $G/R(G)$, where $R(G)$ is the radical of G, then G possesses the R_∞ and S_∞ properties.*

Corollary *Let G be a reductive linear algebraic group over the field F of zero characteristic and finite transcendence degree over $\mathbb{Q}$. If G possesses an automorphism φ with finite Reidemeister number then G is a torus.*

These results can not be extended to groups over a field of non-zero characteristic. It follows from the Lang-Steinberg theorem.

New results on Beauville p-groups

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Given a group G and two elements $x, y \in G$, we denote by $\sum(x, y)$ the union of all conjugates of the cyclic subgroups generated by x, y and xy. Then G is called a *Beauville group* of unmixed type if the following conditions hold:

1. G is a 2-generator group.
2. G has two sets of generators $\{x_1, y_1\}$ and $\{x_2, y_2\}$ such that $\sum(x_1, y_1) \cap \sum(x_2, y_2) = 1$.

In this case, $\sum(x_1, y_1)$ and $\sum(x_2, y_2)$ are said to form a *Beauville structure* for G. It is known that an abelian finite p-group has a Beauville structure if

and only if it is isomorphic to $C_{p^n} \times C_{p^n}$, where $p \geq 5$ and $n \geq 1$. In this talk we will first discuss the conditions under which a 2-generator p-group with "nice power structure" is a Beauville group. These conditions are similar to the conditions for an abelian p-group to be a Beauville group. In particular, this applies to a 2-generator powerful or regular or p-central p-group to be a Beauville group. We next determine which quotients of the Nottingham group over $\mathbb{F}_p$ for an odd prime p are Beauville groups. As a result, we get the first known infinite family of 3-groups admitting a Beauville structure.

This is joint work with Gustavo A. Fernández-Alcober (University of the Basque Country, Spain).

Abelianization of units of modular group rings: the fake degree conjecture and rationality questions

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Let J be a finite dimensional nilpotent algebra over a finite field $\mathbb{F}_q$. Then the set $G = 1 + J$ becomes naturally a finite group. The groups constructed in this way are called *algebra groups*. The group G acts by conjugation on J and this induces a G-action on the dual space $J^* = \mathrm{Hom}_{\mathbb{F}_q}(J, \mathbb{F}_q)$, called the coadjoint action. If $J^p = 0$, there exists an explicit expression of the characters of G in terms of G-orbits in J^*. In particular, one can read the character degrees of G from the square roots of the size of the G-orbits in J^*. I.M. Isaacs conjectured that this was true for any algebra group $G = 1 + J$. This is the so called "Fake degree conjecture". The study of this conjecture will lead us to study the abelianization groups of the form $1 + I_\pi$ where I_π is the augmentation ideal of the group ring $\mathbb{F}_q[\pi]$ for a finite p-group π. Surprisingly, the Bogomolov multiplier of the group π comes into play, and explains why this conjecture is not true in general. We will also explain a nice application to rationality questions in linear algebraic groups.

Tits' Alternative and groups of infinite matrices

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Tits' Alternative says that all finitely generated subgroups of $GL(n, \mathbb{C})$ are either virtually solvable or contain a free (nonabelian) subgroup. In our talk we will consider the group $G = UT(\infty, F)$ of upper unitriangular matrices (indexed by $\mathbb{N}$) over a finite field F. We proved that "almost all" $k-$generator subgroups of G are free of rank k [1]. From the other hand, if F is a two element field, then G contains a first Grigorchuk group of intermediate growth [2]. This shows that G has much more complicated and interested structure than $GL(n, \mathbb{C})$. We survey recent results on the structure of G [3].

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Word fibers in finite p-groups

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Let G be a finite group and let w be a word in k variables. We write $P_w(g)$ the probability that a random tuple $(g_1, \ldots, g_k) \in G^{(k)}$ satisfies $w(g_1, \ldots, g_k) = g$. For non-solvable groups, it is shown in [1] that $P_w(1)$ can take arbitrarily small values as $n \to \infty$. In [4] they prove that for any finite group, G is solvable if and only if $P_w(1)$ is positively bounded from below as w ranges over all words. And G is nilpotent if and only if $P_w(g)$ is positively

bounded from below as w ranges over all words that represent g. Alon Amit conjectured in [2] that in the specific case of finite nilpotent groups and for any word, $P_w(g) \geq 1/|G|$. It is easy to see that it holds for finite abelian groups. Note that it will suffice to prove the conjecture when G is a p-group for some prime p.

We can also consider $N_w(g) = |G|^k \cdot P_w(g)$, the number of solutions of $w = g$ in $G^{(k)}$. Note that N_w is a class function. We prove that if G is a finite p-group of nilpotency class 2, then N_w is a generalized character. What is more, if p is odd, then N_w is a character and for 2-group we can characterize when $N_{x^{2^r}}$ is a character. What is more, we prove the conjecture of A. Amit for finite groups of nilpotency class 2. This result was indepently proved by M. Levy in [3]. Additionally, we prove that for any word w and any finite p-group of class two and exponent p, $P_w(g) \geq 1/|G|$ for $g \in G_w$. As far as we know, A. Amit's conjecture is still open for higher nilpotency class groups. For p-groups of higher nilpotency class, we find examples of words w for which N_w is no longer a generalized character. What is more, we find examples of non-rational words; i.e there exist finite p-groups G and words w for which $g \in G_w$ but $g^i \notin G_w$ for some $(i, p) = 1$.

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Generators for discrete subgroups of 2-by-2 matrices over rational Clifford groups

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In [1], we developed an algorithm to determine generators for discrete subgroups of quaternion algebras over quadratic imaginary extensions of $\mathbb{Q}$ or discrete subgroups of 2-by-2 matrices over quadratic imaginary extensions of $\mathbb{Q}$. These groups act discontinuously on hyperbolic 3-space. The mentioned algorithm constructs a polyhedron containing a fundamental domain in order to find a set of generators.

In this work we imitate this algorithm to Clifford matrices acting discontinuously on hyperbolic n-space. The motivation behind is to get a set of generators for discrete subgroups of 2-by-2 matrices over rational quaternion algebras.

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On the (subgroup) isomorphism problem of group rings

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Let G be a finite group, H a group and R be a commutative ring. The isomorphism problem of finite group rings is the question whether an isomorphism of the group rings RG and RH implies that G and H are isomorphic. It is well known that the answer is not always affirmative. The type of the counterexamples depends of course on the ring R.
In integral group rings the subgroup isomorphism problem is as follows. Let H be a group. Suppose that H occurs as subgroup of the normalized unit

group $V(\mathbb{Z}G)$. Is then H always isomorphic to a subgroup of G ?
Both questions should be considered as questions on the structure of the torsion part of the unit group $U(RG)$ and shoudl be seen in the general problem which properties of a group are reflected by its group rings.
In the talk we give a short survey on the known results (in the case when $R = \mathbb{Z}$ or when R is a field) and present some new results. In the context of the subgroup isomorphism problem the focus lies on the question whether a Sylow like theorem is valid in $V(\mathbb{Z}G)$. It is an open question whether the subgroup isomorphism problem has a positive answer provided the subgroup H is a p - group.

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Sylow numbers and character tables

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Sylow numbers are the set of the numbers of Sylow p-subgroups for all primes p dividing the order of a given finite group G. In 2012 A. Moréto raised the question whether the ordinary character table X(G) of G determines the Sylow numbers of G. In joint work with W. Kimmerle we were able to give a positive answer to the question for finite nilpotent-by-nilpotent or quasinilpotent groups.

If G and H are finite groups with isomorphic integral group rings then it is well known that $\mathrm{X}(G)$ and $\mathrm{X}(H)$ coincide. In the second part of this talk we will show that the integral group ring $\mathbb{Z}G$ of a finite soluble group G determines the Sylow numbers of G.

Critical groups isospectral to $U_3(3)$

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All groups in this talk are finite. The *spectrum* $\omega(G)$ of a group G is the set of its element orders. By a *section* of G we mean a quotient group H/N, where $N, H \leq G$ and $N \trianglelefteq H$. Groups G and H are called *isospectral*, if $\omega(G) = \omega(H)$. Let ω be a subset of natural numbers. Following [1], we call a group G *critical with respect to* ω (or ω-*critical*), if ω coincides with the spectrum of G and does not coincide with the spectrum of any proper section of G.

If a simple group L has infinitely many groups isospectral to L, then it is important to study critical groups isospectral to L. In [2],[3] the complete description is given of critical groups isospectral to non-abelian simple alternating and sporadic groups and also the special linear group $SL_3(3)$.

In this work we study groups critical with respect to the spectrum of the projective special unitary group $U_3(3)$. In particular, we prove the following

Theorem. *Let G be a group isospectral to $U_3(3)$ that contains a normal subgroup N, such that $G/N \simeq PGL_2(7)$. Then N is a 2-group and every G-chief factor of N is isomorphic to a 6-dimensional module of the group $PGL_2(7)$. Also $G = NH$ for some subgroup $H \simeq PGL_2(7)$. If in addition G is critical with respect to $\omega(U_3(3))$, then $|N| = 2^6$.*

Moreover, H has a representation $\langle a, b, c \mid a^2 = b^3 = c^2 = (ab)^7 = (ac)^2 = (bc)^2 = [a, b]^4 = 1\rangle$ and if we regard N as a vector space over $GF(2)$ then a base of N can be chosen in such a way that the action of H on N is defined by the following matrices:

$$a \sim \begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & 1 \end{pmatrix}, \; b \sim \begin{pmatrix} \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \end{pmatrix}, \; c \sim \begin{pmatrix} \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\ 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot \end{pmatrix}.$$

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Where are the groups of intermediate growth?

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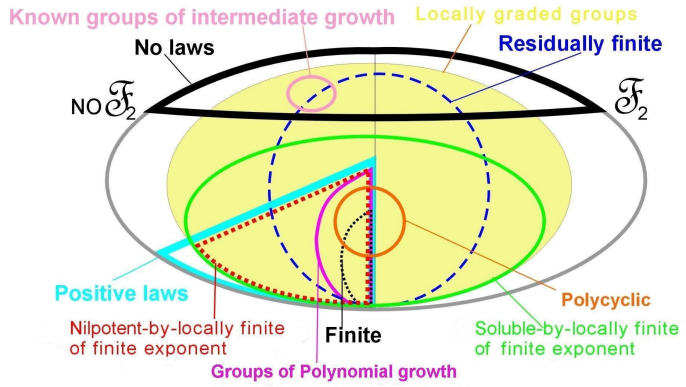
Different classes of groups have their "countries" on the planet Groupland. Till 2004 the groups of intermediate growth lived among the residually finite groups. After 2004 their territory was extended. The answer to the question of A.Mann *whether a group of intermediate growth can satisfy a non-trivial law* will produce some specific changes.
We can prove the following

Theorem A group of intermediate growth satisfying a positive law exists only if there exists a simple non locally finite group of intermediate growth satisfying this law.



Prime graph question for torsion units in group rings of 4-primary groups

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Let G be a finite group and $V(\mathbb{Z}G)$ the group of normalized units in the integral group ring of G. Let p and q be different primes. The Prime Graph Question (PQ), a weak form of the Zassenhaus Conjecture, asks whether G must contain an element of order pq, if this is the case for $V(\mathbb{Z}G)$. By a reduction of W. Kimmerle and A. Konovalov it is enough to investigate this problem for almost simple groups. I am going to present results on (PQ) for 4-primary groups, i.e. groups whose order has 4 pairwise different prime divisors. The methods involved are the HeLP-method (presented in two other talks at this conference) and the so called Lattice-method. Some special emphasis will be placed on algorithmic aspects of the Lattice-method.

Infinite generalizations of Hamming spaces and their isometry groups

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It will be considered a few constructions of infinite metric spaces. All of them can be regarded as generalizations of finite Hamming spaces for the infinite dimentional case. In particular, it will be considered a continuum family of compact separable subspaces of the Besicovitch space, naturally parameterized by supernatural numbers. It is well known that the isometry group $IsomH_n$ of the Hamming space H_n is the hyperoctahedral group W_n. It will be described the isometry groups of the spaces under consideration. Connections with hyperoctahedral groups will be discussed.

Computational aspects in Steiner loops

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The fact that all elements different from 1 of a Steiner loop have order 2 makes it attractive to apply formal grammars in this variety. In my talk some consequences of this approach will be discussed.

Finite nearrings on split metacyclic groups

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The determination of the non-abelian finite p-groups which are the additive groups of local nearrings is an open problem [1].

The aim of the talk is to discuss this question. Furthermore, we describe all possible types of split metacyclic groups which are the additive groups of finite local nearrings.

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[1] Fiegelstok S. Additive groups of local nearrings, Comm. Algebra, **34** (2006), P. 743–747.

Finite local nearrings on non-abelian p-groups with cyclic commutator subgroups

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Nearrings are a generalization of rings in the sense that the addition in nearrings need not be commutative and only one distributive law holds. Basic definitions and many results concerning nearrings can for instance be found in Pilz [1].

We study possible types of finite non-abelian 2-generated groups with cyclic commutator subgroup, for odd primes p, which are the additive groups of local nearrings.

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Groups of half-automorphisms of loops.

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I will discuss the properties of automorphic Moufang loops and will speak about the half-automorphisms of loops. It will be shown that for finite and for free automorphic Moufang loops half-automorphisms are automorphisms or anti-automorphisms. The group of half-automorphisms of a code loop will be presented.

On sets of grenerators of finite groups

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All groups considered here are finite. By $\Phi(G)$ we denote the Frattini subgroup of a group G. There exist several notions of independence in algebra, also in group theory. Here a subset $X \subseteq G$ is said to be:

- *g-independent* if $\langle Y, \Phi(G)\rangle \neq \langle X, \Phi(G)\rangle$ for every proper subset $Y \subset X$;
- a *g-base* of G, if X is a g-independent and $G = \langle X\rangle$.

During the talk we are going to present some older and new results concerning finite groups with properties analogous to that of vector space. We say that group G

- has *property* $\mathcal{B}$ (is a $\mathcal{B}$-group) if every two g-bases of G have the same cardinality;
- has the *embedding property* if every g-independent subset of G is contained in a g-base of G;
- is *matroid* if G has property $\mathcal{B}$ and the embedding property;
- has the *basis property*, if every subgroup of G is a $\mathcal{B}$-group.

Groups with such properties are studied from several years (see [1],[2], [6 – 8]). Classes of groups with property $\mathcal{B}$, the basis property and matroid grups are described (see [1], [3], [6], [7]). These classes are rather narrow. Thus we proposed the modification of these notions which give wider classes of groups. If only generators of prime power orders are considered, then an analog of property $\mathcal{B}$ was denoted in [5] by $\mathcal{B}_{pp}$ *property* and an analog of the basis property was named the *pp-basis property*.

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Central idempotents in group graded rings

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The well known theorem of Burns says the support subgroup of every central idempotent of group ring RG is a finite normal subgroup of G. We will show central idempotents in group-graded rings with infinite group generated by this support. The support groups of central idempotents in group-graded rings are finite when we have some restrictions on groups e.g. G is locally finite by locally indicable or when we have some restrictions on ring e.g. the gradation is strong.

Locally finite groups of automorphisms of atomless countable Boolean algebra

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In the talk we characterize the locally finite groups of automorphisms of atomless countable Boolean algebra $\mathcal{B}$. We introduce the concept of LD-automorphisms and characterize the groups of LD-automorphisms, which are defined by increasing series of finite subalgebras of $\mathcal{B}$.

Maximal subgroups of branch groups

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Stemming from the Burnside Problem, branch groups are an interest in their own right. We look at questions concerning branch groups, in particular the index of their maximal subgroups.

On NA-varieties of groups

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We say that a variety of groups is **an NA-variety** if every nilpotent group in this variety is abelian and an identity $w(x,y) \equiv 1$ is **an NA-identity** if a variety of groups satisfying $w(x,y) \equiv 1$ is an NA-variety. In the talk we describe properties of NA-varieties, give conditions for an identity to be an NA-identity and show that many known from literature classes of varieties are NA-varieties.

Proposition 1. *Let $\mathfrak{U}$ be a variety of groups. Then the following conditions are equivalent.*
(1) *$\mathfrak{U}$ is an NA-variety,*
(2) *if G is any group in $\mathfrak{U}$ then $G' = \gamma_3(G)$,*
(3) *if G is any metabelian group in $\mathfrak{U}$ then $G' = \gamma_3(G)$,*
(4) *if G is any finite group in $\mathfrak{U}$ then $G' = \gamma_3(G)$,*
(5) *if G is any two-generator group in $\mathfrak{U}$ then $G' = \gamma_3(G)$,*
(6) *if G is any two-generator metabelian group in $\mathfrak{U}$ then $G' = \gamma_3(G)$,*
(7) *if $G = F_2(\mathfrak{U})$ is a two-generator free group in $\mathfrak{U}$ then $G' = \gamma_3(G)$.*

We say that a finite group G is **an A-group** if every its Sylow's subgroup is abelian.

Theorem 1 (A. Yu. Ol'shanskii, cf. [1]**.** *] Let $\mathfrak{U}$ be a variety of groups. Every group in $\mathfrak{U}$ is residually finite if and only if $\mathfrak{U}$ is generated by a finite A-group.*

Theorem 2. *A variety $\mathfrak{U}$ is an NA-variety if and only if every finite group in $\mathfrak{U}$ is an A-group.*

Theorem 3. *Let $\mathfrak{U}$ be an NA-variety. Then*
(1) *Every finitely generated metabelian group in $\mathfrak{U}$ is abelian-by-(finite A-group).*
(2) *Every finitely generated residually finite group in $\mathfrak{U}$ is abelian-by-(finite A-group).*

Theorem 4. *$\mathfrak{U}$ is an NA-variety if and only if there exists an identity $w(x,y) = w(x,y) = [x,y]^{f(x,y)x^{-n}y^{-m}}\xi \equiv 1$ where $f(1,1) = 0$ such that every group in $\mathfrak{U}$ satisfies it.*

Theorem 5. *The set of NA-varieties form a lattice.*

Let $\mathfrak{A}_n$ denote a variety of abelian groups of exponent n.

Theorem 6. *A variety $\mathfrak{A}_n\mathfrak{A}_m$ is an NA-variety if and only if* $\mathrm{nwd}(n,m)=1$.

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The constructions of amenable groups based on automata over a changing alphabet

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We investigate the notion of the nearly finitary group introduced in [3] and use the recently obtained results ([1,4]) concerning amenability of groups generated by automorphisms of a spherically homogeneous rooted tree. We show that every nearly finitary group is amenable and apply this result to two group constructions introduced in [2,3]. The two constructions have a natural description in the language of an automaton over the changing alphabet $X=(X_i)_{i\geq 1}$, where each X_i is a finite and non-empty set of letters. By these constructions, we prove amenability for some finitely generated, weakly branch groups containing non-Abelian free semigroups, which are dense in inverse limits

$$\wr_{i=1}^{\infty} G_i := \varprojlim_{i\to\infty} (\dots (G_i \wr_{X_{i-1}} \dots \wr_{X_3} G_3) \wr_{X_2} G_2) \wr_{X_1} G_1 \qquad (1)$$

of iterated permutational wreath products of finite, transitive permutation groups (G_i, X_i), $i \geq 1$. The two constructions are universal: the first works with an arbitrary sequence $(G_i, X_i)_{i\geq 1}$ of non-Abelian, simple groups and

the second construction works with an arbitrary sequence of non-trivial, Abelian groups such that the corresponding inverse limit is topologically finitely generated. In particular, we obtain the following results

Theorem 1. *Let $X = (X_i)_{i\geq 1}$ be a changing alphabet and let $(H_i, X_i)_{i\geq 1}$ be an arbitrary sequence of non-Abelian, simple, transitive permutation groups. Then the inverse limit $\wr_{i=1}^{\infty} H_i$ is the topological closure of a 2-generated, amenable, nearly finitary group G which is non-elementary amenable. The group G is generated by an automaton $\mathcal{A}$ over the alphabet X, which is equipped with three states. One of the states is neutral (i.e. induces the trivial automorphism) and the other two states generate a free semigroup. The automaton $\mathcal{A}$ is self-similar and of branch type over the sequence of commutator subgroups. In particular, the group G has an exponential growth and it is an example of a weakly branch group.*

Theorem 2. *Let $X = (X_i)_{i\geq 1}$ be a changing alphabet and let $(A_i, X_i)_{i\geq 1}$ be an arbitrary sequence of non-trivial, Abelian, transitive permutation groups such that the topological rank ρ of the infinite cartesian product $\prod_{i=1}^{\infty} A_i$ is finite. Then the inverse limit $\wr_{i=1}^{\infty} A_i$ is the topological closure of an amenable, 2ρ-generated, weakly branch group G such that the commutator subgroup G' is a nearly finitary group. The group G is generated by a self-similar automaton $\mathcal{B}$ over the alphabet X. The automaton $\mathcal{B}$ is equipped with 2ρ states $a_1, \ldots, a_\rho$, $b_1, \ldots, b_\rho$ such that both the group $gp(a_1, \ldots, a_\rho)$ and the group $gp(b_1, \ldots, b_\rho)$ are the free Abelian group of rank ρ. Moreover, the semigroup $sgp(a_1, \ldots, a_\rho, b_1, \ldots, b_\rho)$ is the free product of the semigroups $sgp(a_1, \ldots, a_\rho)$ and $sgp(b_1, \ldots, b_\rho)$.*

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2-Absorbing Commutative Semigroups

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Throughout the talk S denotes a commutative semigroup with 0. We say that S is 2-absorbing if for arbitrary elements $s_1, s_2, s_3 \in S$ satisfying $s_1 s_2 s_3 = 0$, there are $1 \leq i \neq j \leq 3$ such that $s_i s_j = 0$. We say that S is *strongly 2-absorbing* if for arbitrary ideals I_1, I_2, I_3 of S satisfying $I_1 I_2 I_3 = 0$, there are $1 \leq i \neq j \leq 3$ such that $I_i I_j = 0$. It is clear that strongly 2-absorbing semigroups are 2-absorbing, but by an appropriate simple example we show that the converse does not hold.

We show that a conjecture concerning n-absorbing rings stated by Badawi and Anderson [1] holds for rings with torsion-free additive groups.

References:

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Non-trivial group operations

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Some word $u(x, y)$ defines the group operation in a group G, if the set G with function $x \circ y = u(x, y)$ is also a group and moreover there exists a word $v(x, y)$ such that in group $(G, \circ)$ $xy = v_\circ(x, y)$. Of course in every group, $x \circ y = xy$ and $x \circ y = yx$ are group operations, called trivial. If the groups G and $(G, \circ)$ are isomorphic, then isomorphism is called a weak automorphism of group G.

In free groups there is no non-trivial group operations. In 2-nilpotent groups of finite exponent n of commutator G', all words of the form $x \circ y = xy[x, y]^k$, where $\mathrm{nwd}(2k + 1, n) = 1$ are group operations ([1]). In groups of finite exponent n, some group operations has the power form $x \circ y = (x^k y^k)^m$, where $k \cdot m = 1 \bmod \exp G$ ([2]). In nilpotent groups of class 3 and 4 some group operations were presented by A. Street ([3]).

The problems are to find all groups operations for some class of groups. For Coxeter groups and some generalizations all groups operations has power form. In [1] and [3] there is examples of non-power group operation in finite and infinite groups.

For power group operations it is obvious that the groups G and $(G, \circ)$ are isomorphic. For relatively-free groups with some conditions, all group operations leads to isomorphic groups.

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